Pareto Efficiency

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The Two Welfare Theorems of Welfare Economics

1. A competitive equilibrium is a *Pareto-efficient* allocation.
2. Any Pareto-efficient allocation can be achieved as a competitive equilibrium with an appropriate redistribution of the initial wealth.

Pareto Efficiency

- An *allocation* is a distribution of goods and services as an outcome of market transactions.
- An allocation is *Pareto-efficient* if no agent can improve her welfare without reducing other agents' welfare.

A Pure Exchange Economy

- There are two agents, A and B (a employee and an employer, Japan and USA, etc.)
- There are two goods, 1 and 2. The total amounts of goods 1 and 2 are normalized to one.
- Let (y_1, y_2) represent the initial wealth of agent A. Then the initial wealth of agent B is represented by $(1-y_1, 1-y_2)$.
- The economy is illustrated as a rectangular, called the “Edgeworth Diagram”.

The budget constraint of agent A is

$$p_1 x_{1A} + p_2 x_{2A} = p_1 y_{1A} + p_2 y_{2A}$$

where (p_1, p_2) is the price vector of good 1 and good 2.

The budget constraint can be rewriten as

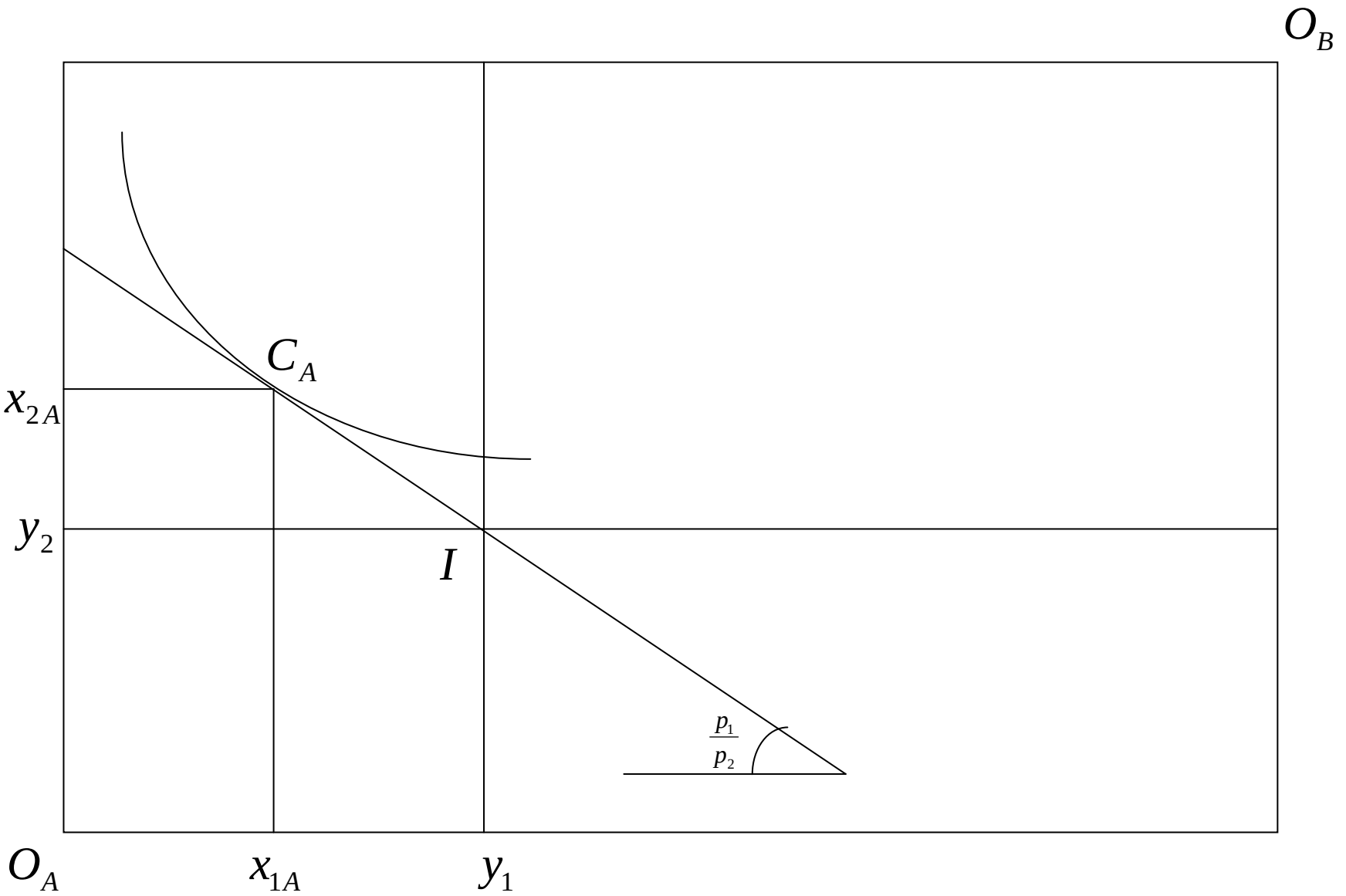
$$x_{2A} = \left(-\frac{p_1}{p_2}\right) x_{1A} + \left(\frac{p_1}{p_2}\right) y_{1A} + y_{2A}$$

Therefore the budget line runs through point I, and has a slope of $\left(-\frac{p_1}{p_2}\right)$.

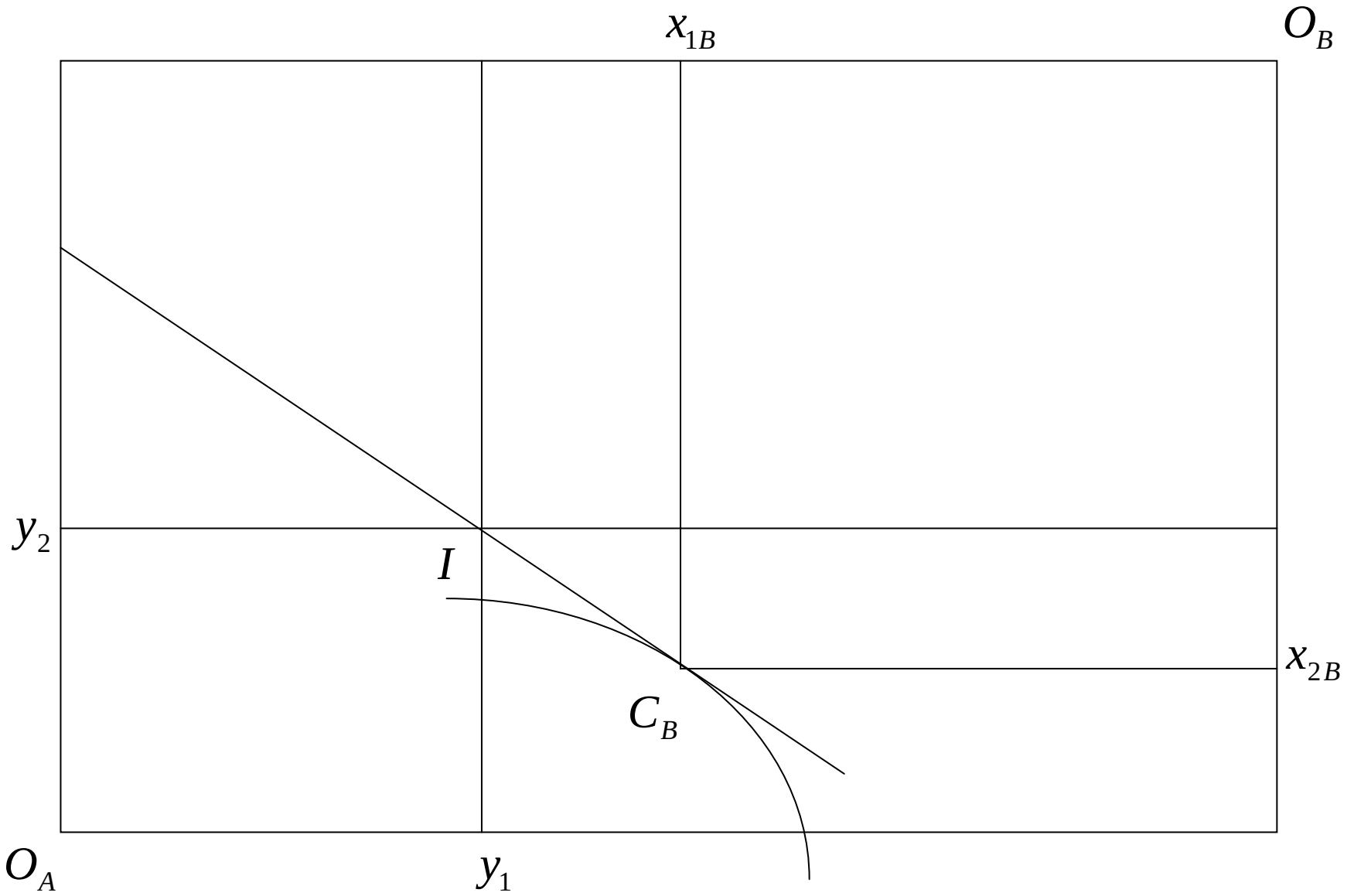
At any equilibrium, the market must clear in the sense that

$$x_{1A} + x_{1B} = 1, \quad x_{2A} + x_{2B} = 1.$$

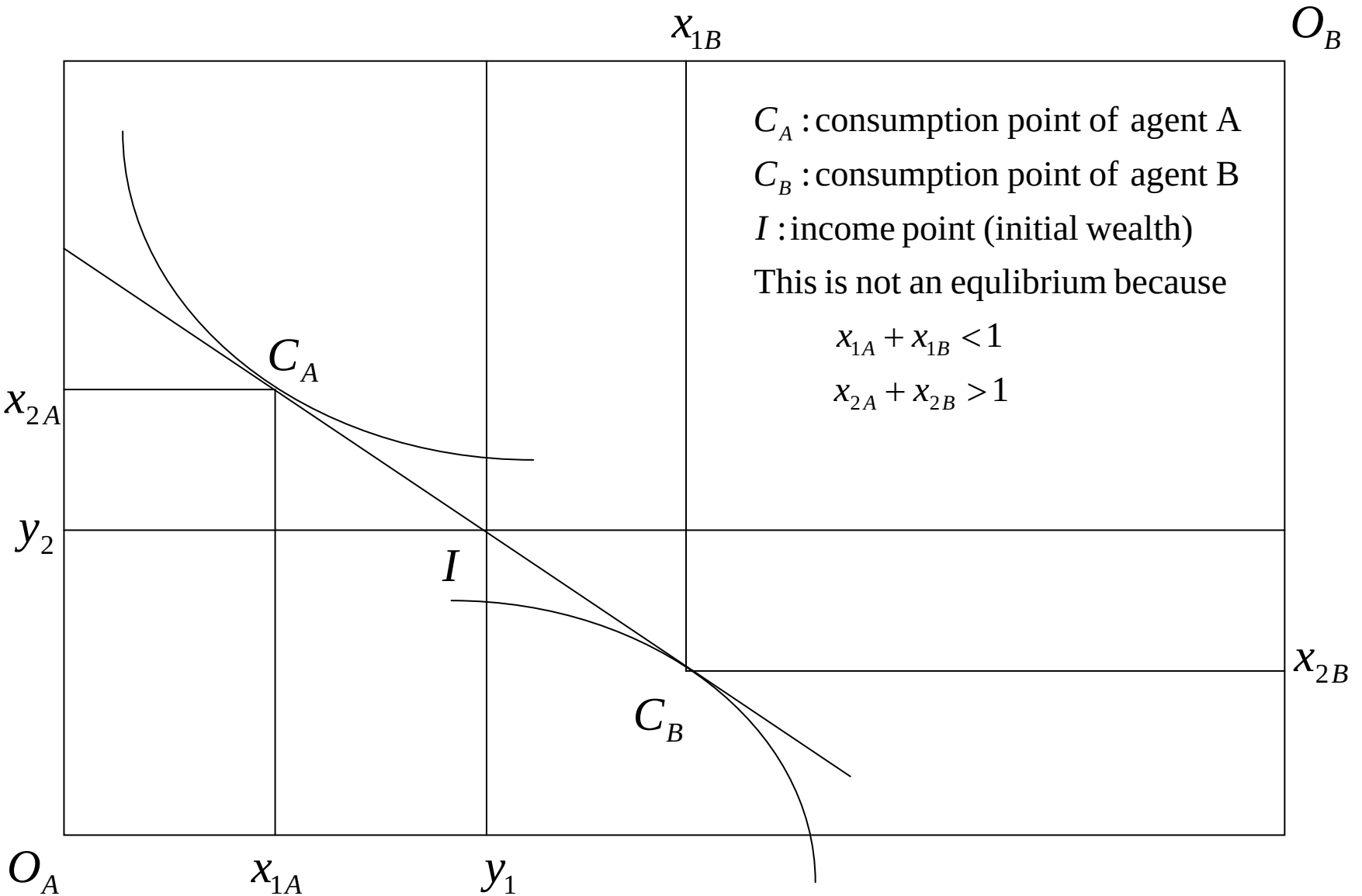
The decision of agent A.



The decision of agent B.



This is not an equilibrium, with too much of good 1 and too little of good 2.



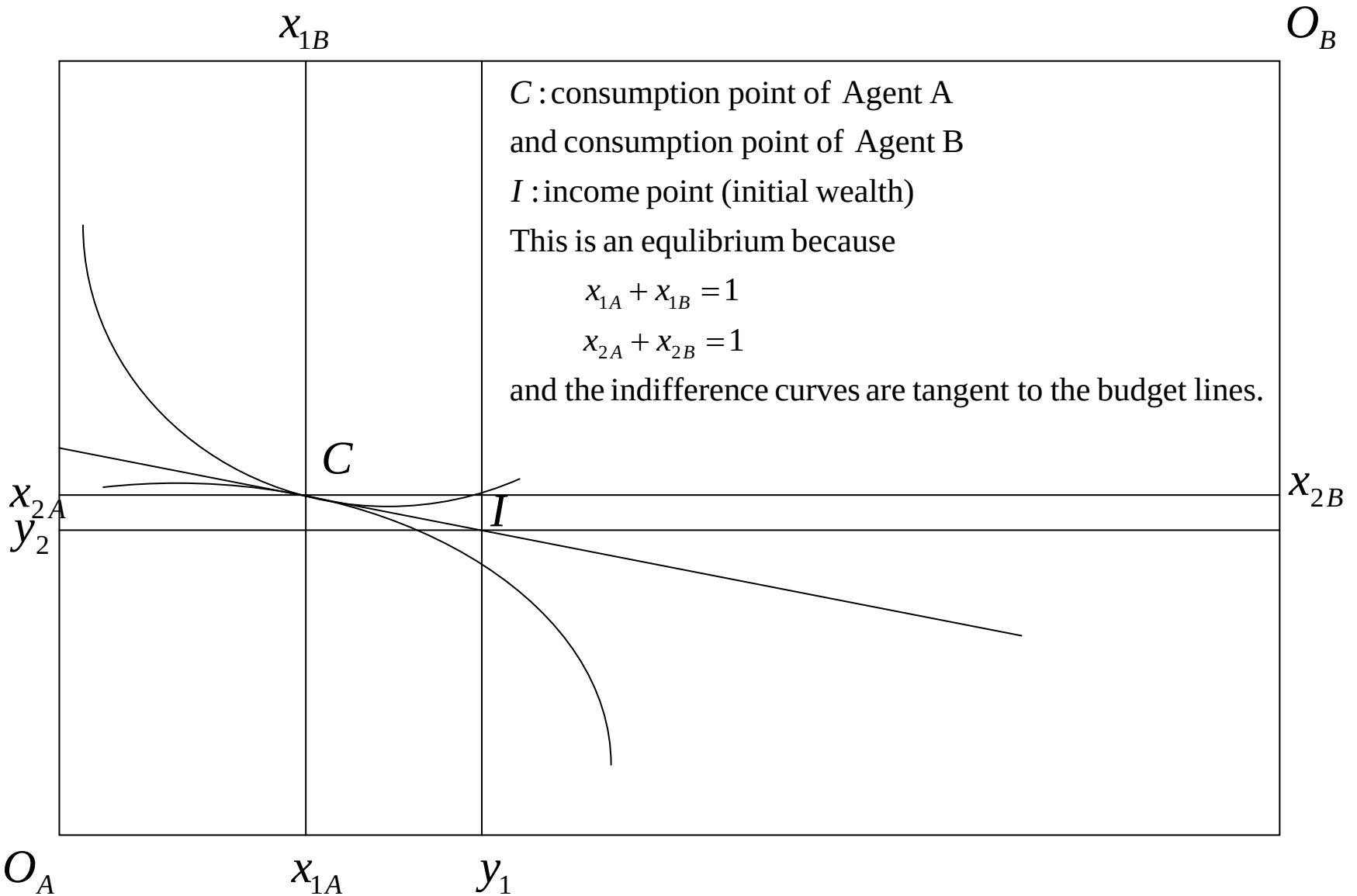
Why doesn’t the market clear?

- Because the price of good 1 is two high, and the price of good 2 is two low.
- If the price of good 1 is too high, there is excess supply for good 1.
- If the price of good 2 is too low, there is excess demand for good 2.

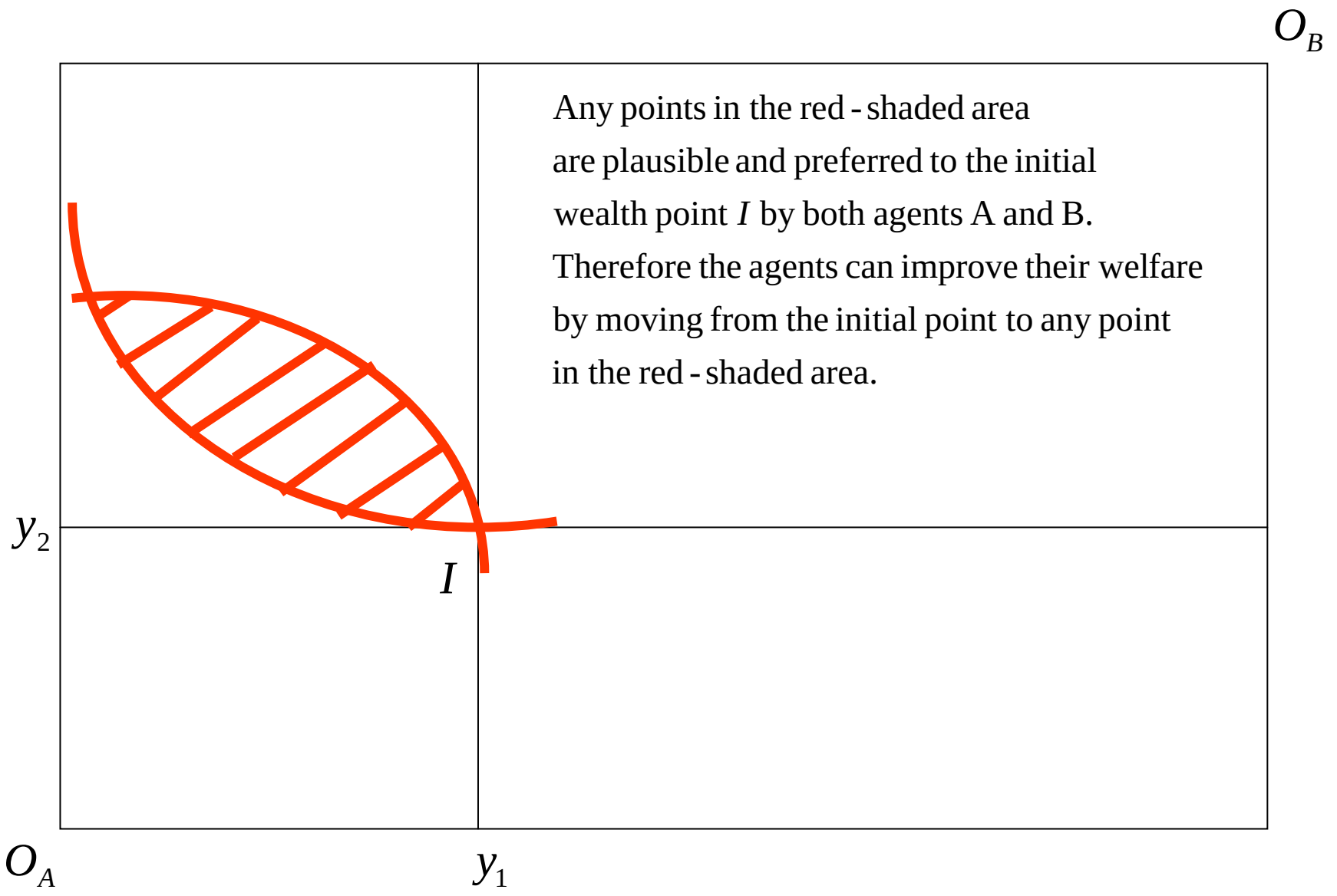
$$x_{1A} + x_{1B} < 1$$

$$x_{2A} + x_{2B} > 1$$

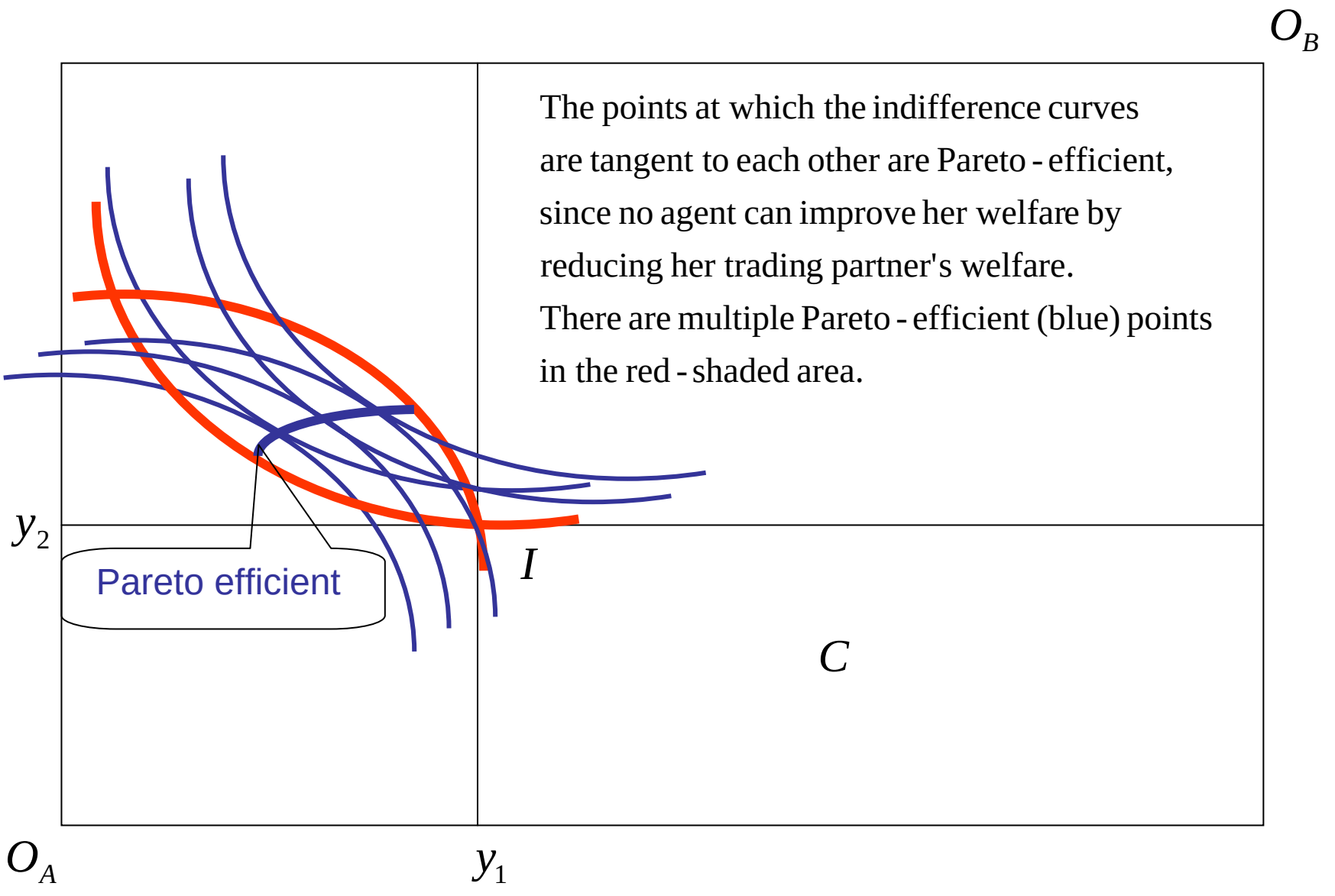
This is an equilibrium because the market clears.



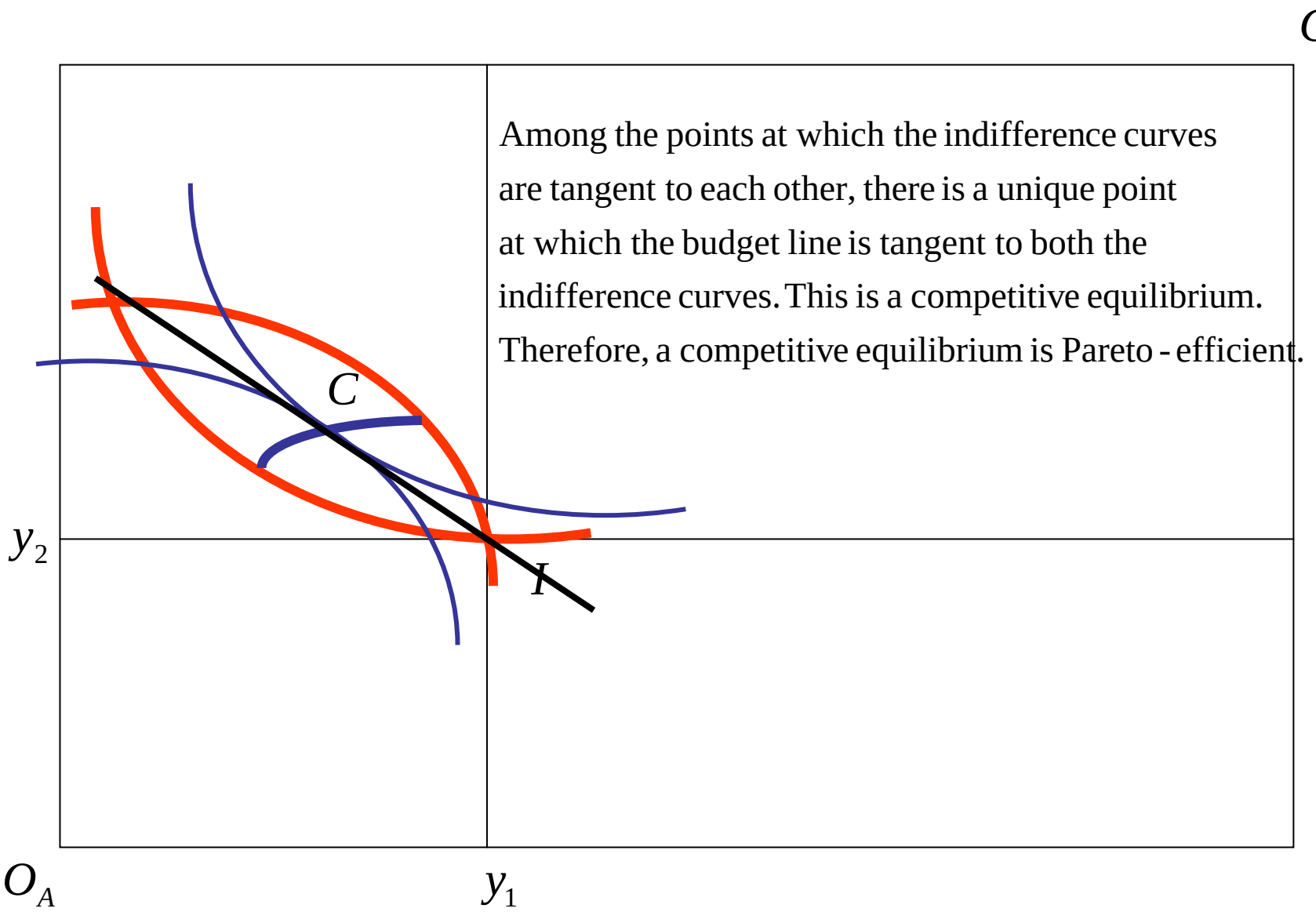
The initial point *I* can be *improved upon* by any points in the red-shaded area.



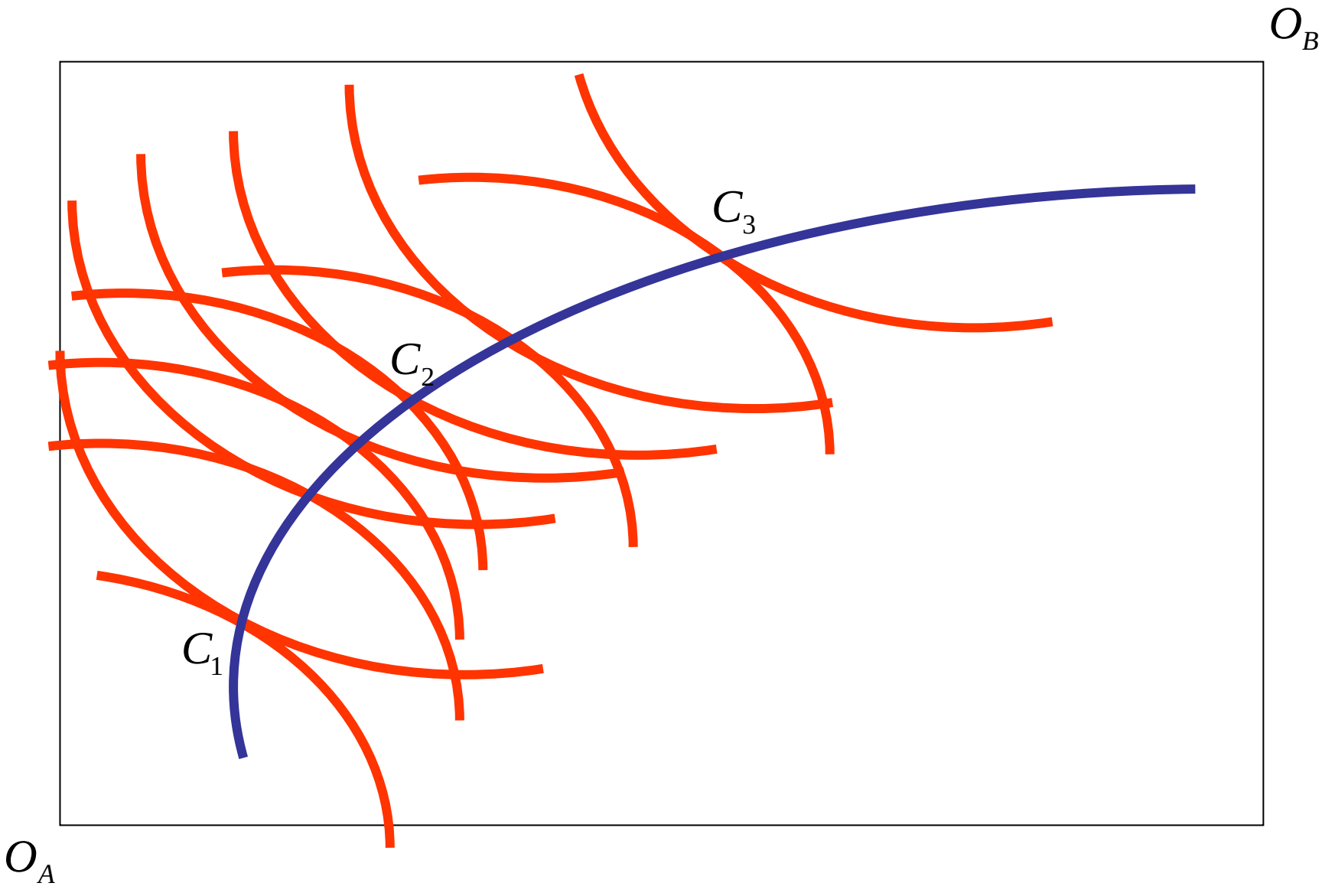
There are multiple Pareto-efficient points in the red-shaded area.



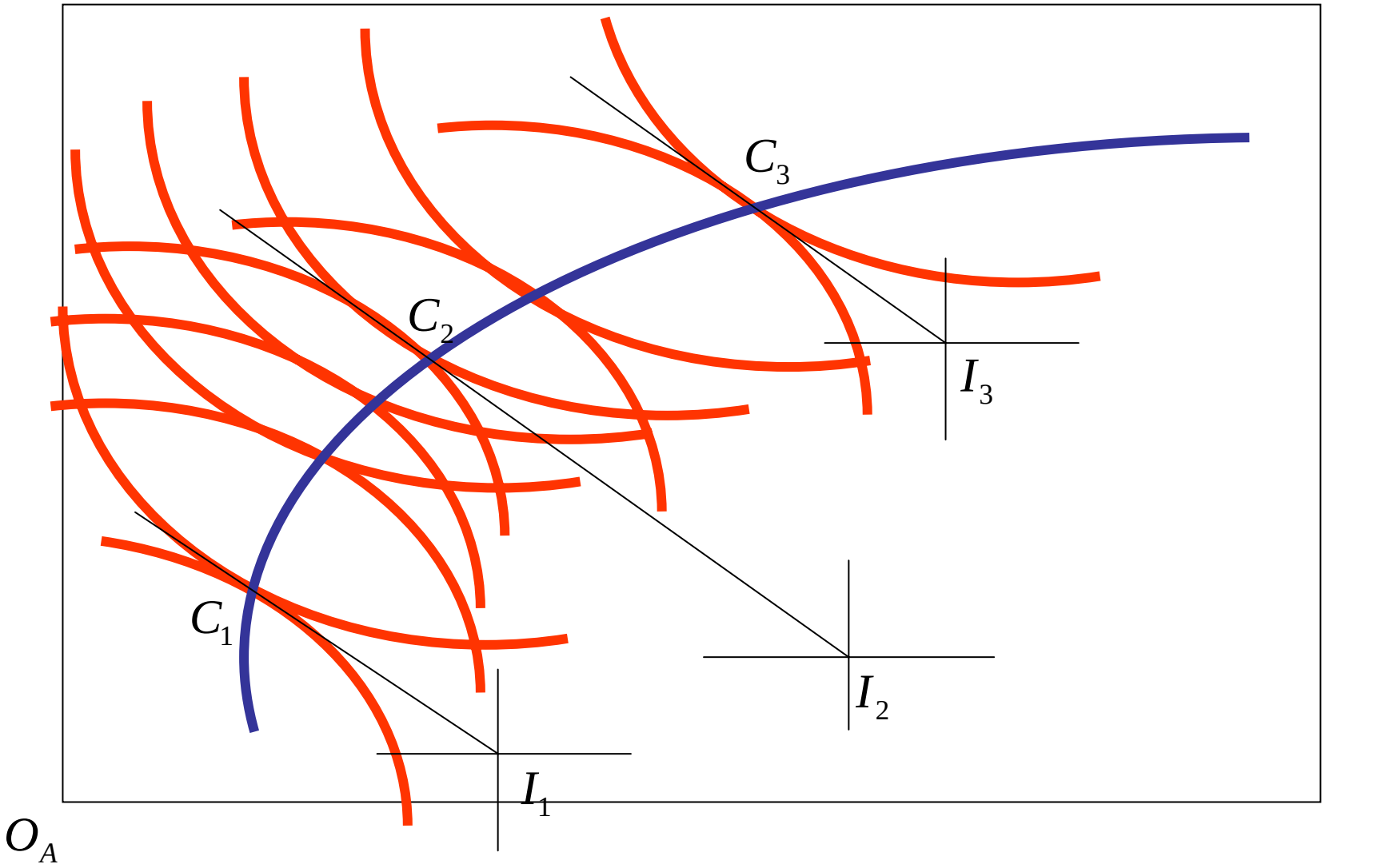
The First Welfare Theorem.



There are multiple Pareto-efficient points.



The Second Welfare Theorem.
Any Pareto-efficient point can be achieved as a competitive equilibrium.



Consider three Pareto - efficient points, $C_1, C_2,$ and C_3. These points can be achived as competitive equilibria from the initial welath, $I_1, I_2,$ and I_3, respectively.

Assume that the initial wealth point is I_1. Then the Pareto - efficient allocation which can be achieved from this initial point is C_1. However, the government wants to obtain point C_2. Can it do that?

Yes. It's easy. The government can move the initial point from I_1 to I_2 by some tax and subsidy policy.
Then the market competition results in the Pareto - efficient allocation, C_2.

Remarks

1. In the Second Welfare Theorem, it is assumed that the government can change the initial wealth by a *lump-sum* tax and subsidy, without causing any *distortion* of the individual decisions.
2. However, it is hard to cause no distortion in the real economy. In most cases, the government can change the initial wealth distribution only by some distortionary tax such as (labor) income tax.

Distortion

- Any individual decision can be distorted by some distortionary tax such as labor income tax.
- The higher income tax discourages workers from working harder. Then the distortionary tax reduces the aggregate amount of income.