Production and Investment

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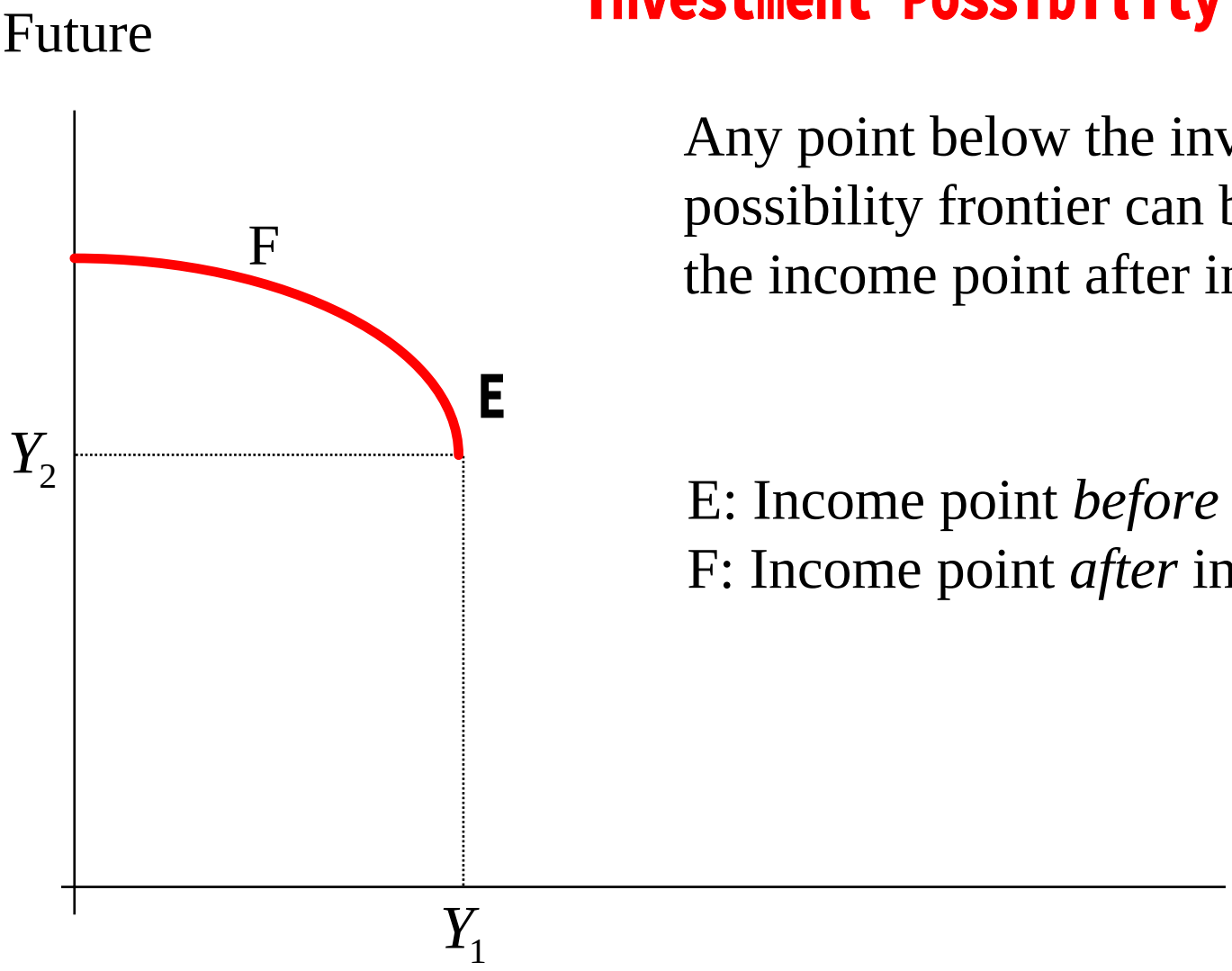
Investment Creates More Opportunities

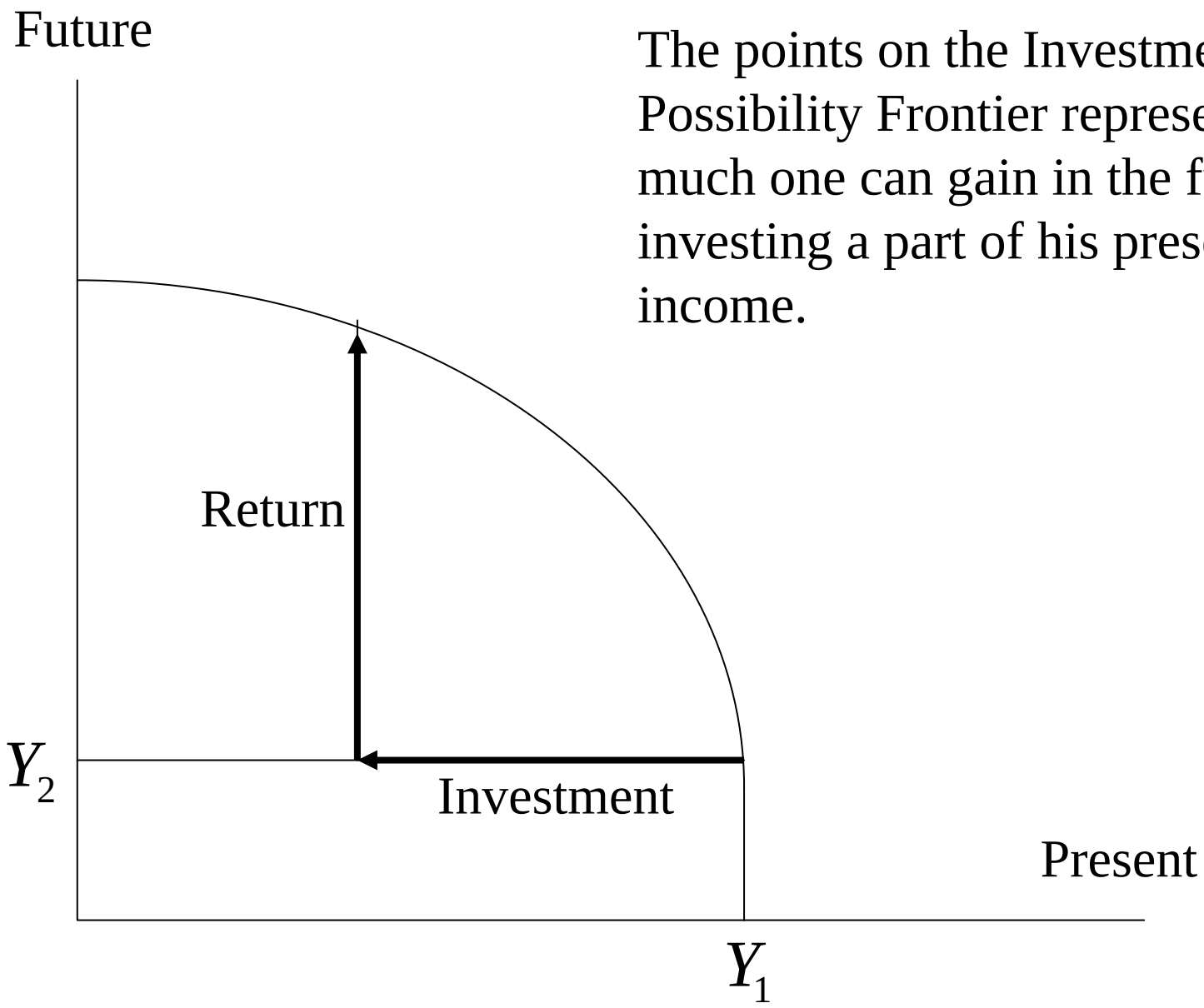
- In the basic model, the income pattern (present - future) is given, and the agent cannot change her/his future income.
- However, the agent can increase her/his future income by investing a portion of her/his present income in a project to increase the future income.

Investment Possibility Frontier

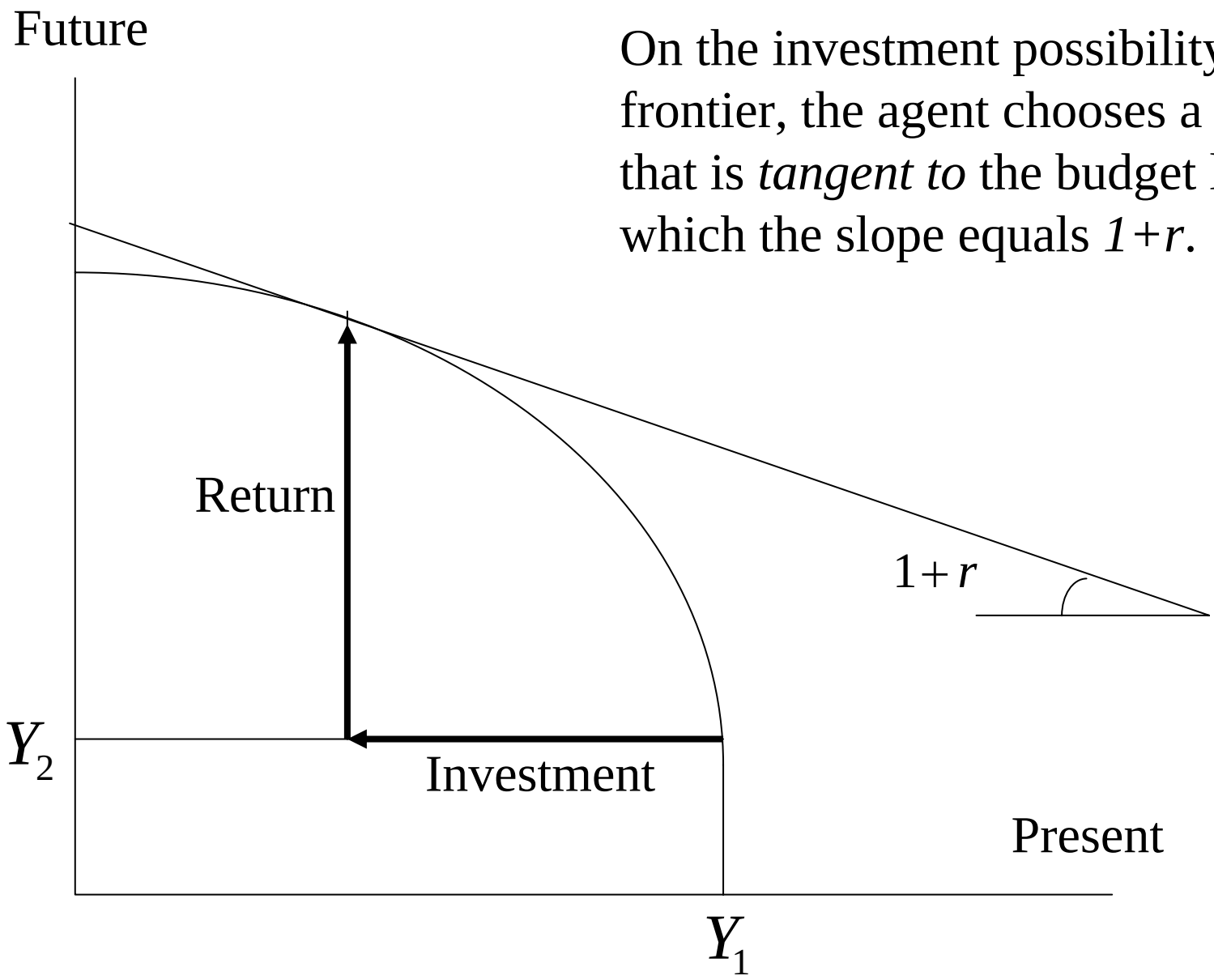


Any point below the investment possibility frontier can be chosen as the income point after investment.

E: Income point *before* investment
F: Income point *after* investment



The points on the Investment Possibility Frontier represents how much one can gain in the future by investing a part of his present income.



On the investment possibility frontier, the agent chooses a point that is *tangent to* the budget line, of which the slope equals $1+r$.

The Investment Decision

The agent spends I in the present period, and obtains the outcome $F(I)$ in the future period, which must be discounted by the interest rate r.

The agent problem is to maximize the net gain of investment,

$$\frac{F(I)}{1+r} - I$$

where $F(I)$ denote the output of investment.

The first-order condition for the maximization is

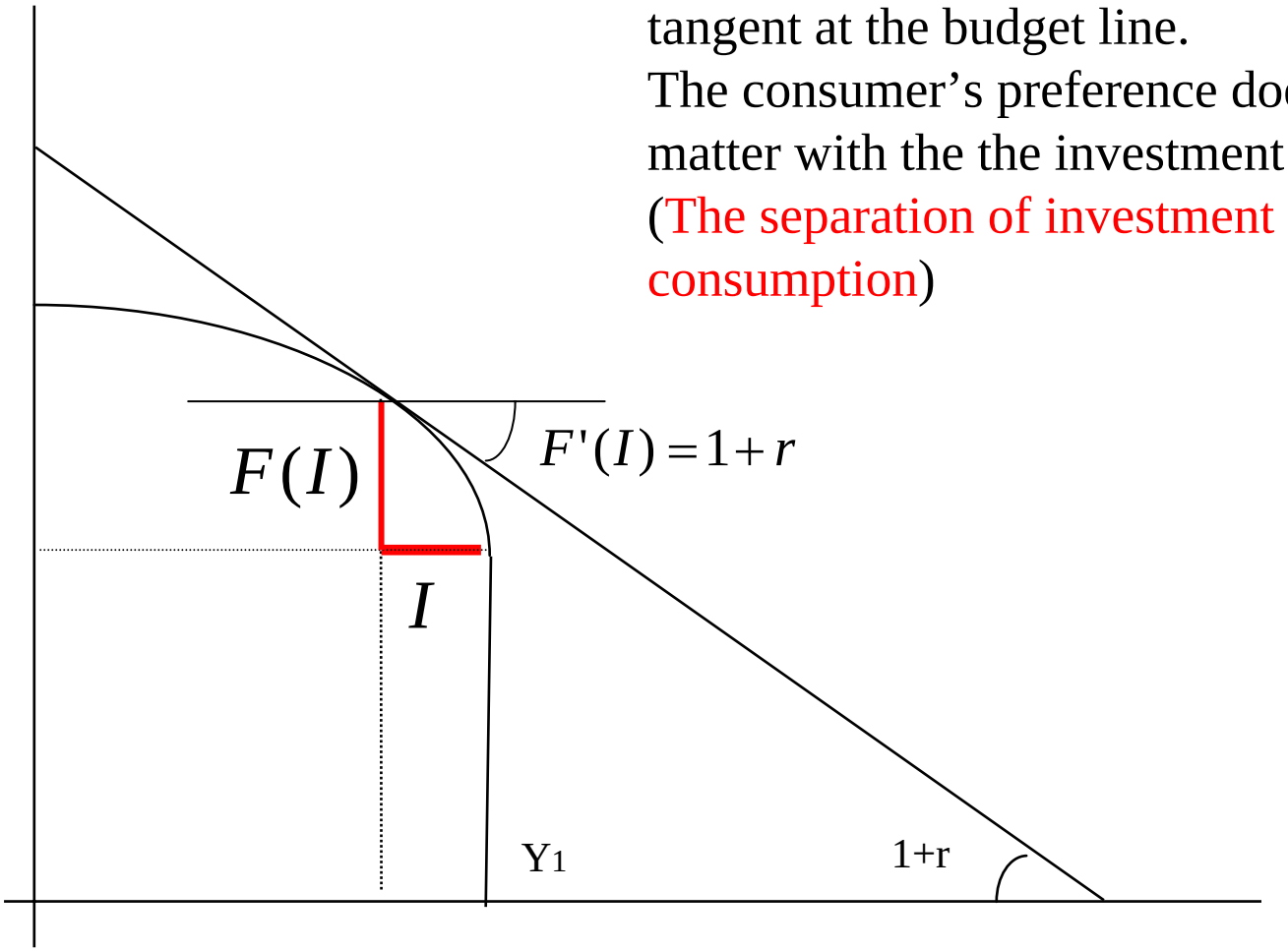
$$F'(I) = 1 + r$$

In other words, the marginal return of investment equals the gross interest rate (one plus interest rate).

The investment decision is independent of the consumption/saving decision.

Future

The investment is determined such that the investment possibility frontier is tangent at the budget line.
The consumer's preference does not matter with the the investment decision.
(The separation of investment from consumption)



Present

A More Mathematical Treatment: The Neoclassical Production Function

Production Function is a relationship between inputs and outputs.

There are *n* inputs and *m* outputs.

$$(y_1, y_2, y_3, \ldots y_m) = F(x_1, x_2, x_3, \ldots x_n)$$

The most important example is :

$$y = F(K, L)$$

where K denotes capital and L denotes labor.

The production function shows constant returns to scale if

$$F(\lambda K, \lambda L) = \lambda F(K, L), \quad \text{where } \lambda > 0.$$

The Firm

The purpose of a firm is to maximize its profit, which eventually belongs to households. The firm's profit is

$$\pi = pF(K,L) - rK - wL$$

where w is the wage rate and r is the interest rate.

Assume that π is differentiable in K and L.

Then the first order condition of the profit maximization is

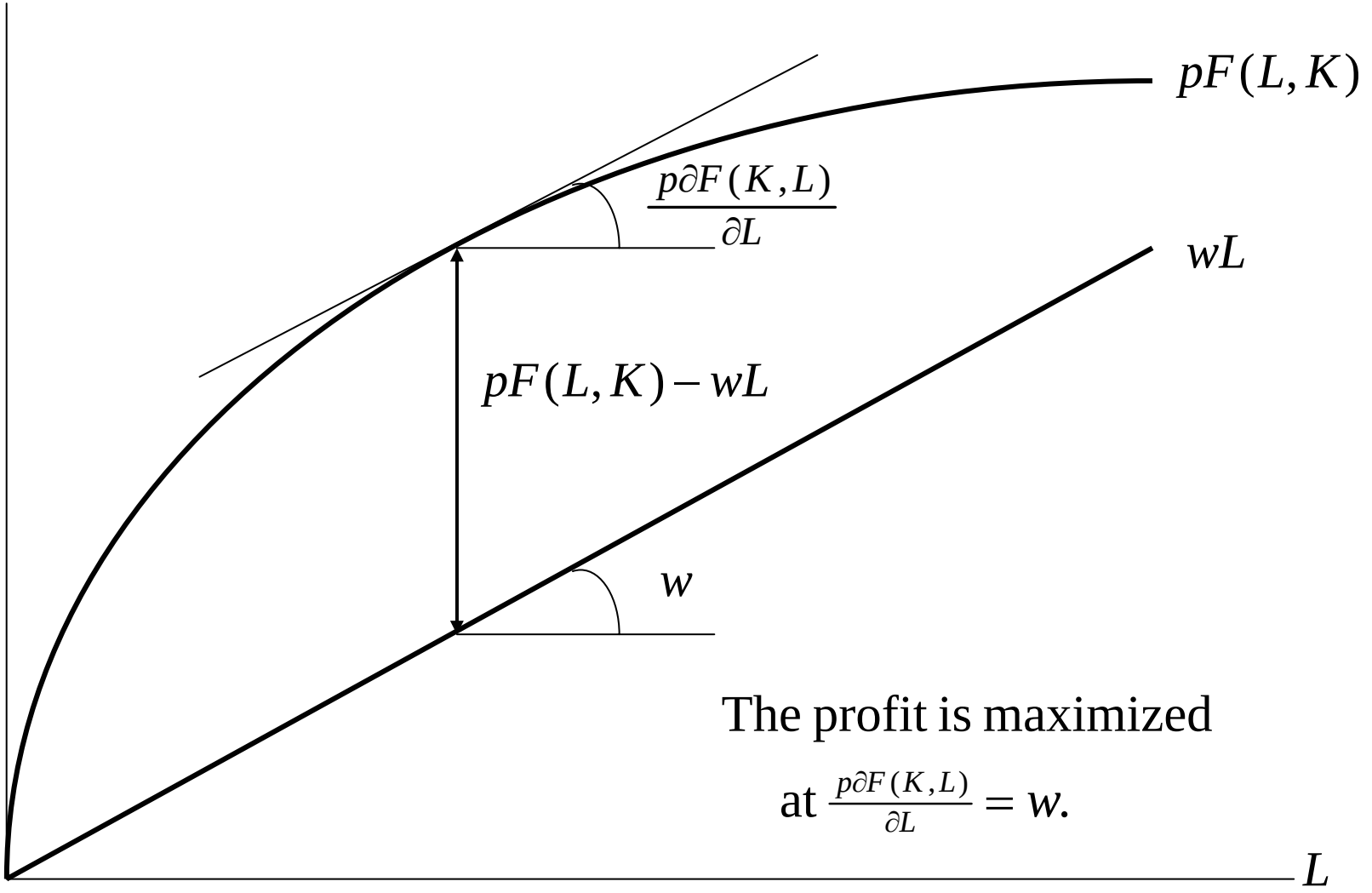
$$\frac{p\partial F(K,L)}{\partial K} = r, \quad \frac{p\partial F(K,L)}{\partial L} = w$$

In other words :

The marginal productivity of capital equals the interest rate.

The marginal productivity of labor equals the wage rate.

The Margin Principle



The profit is maximized

at $\frac{p\partial F(K,L)}{\partial L} = w.$

Country = Agent

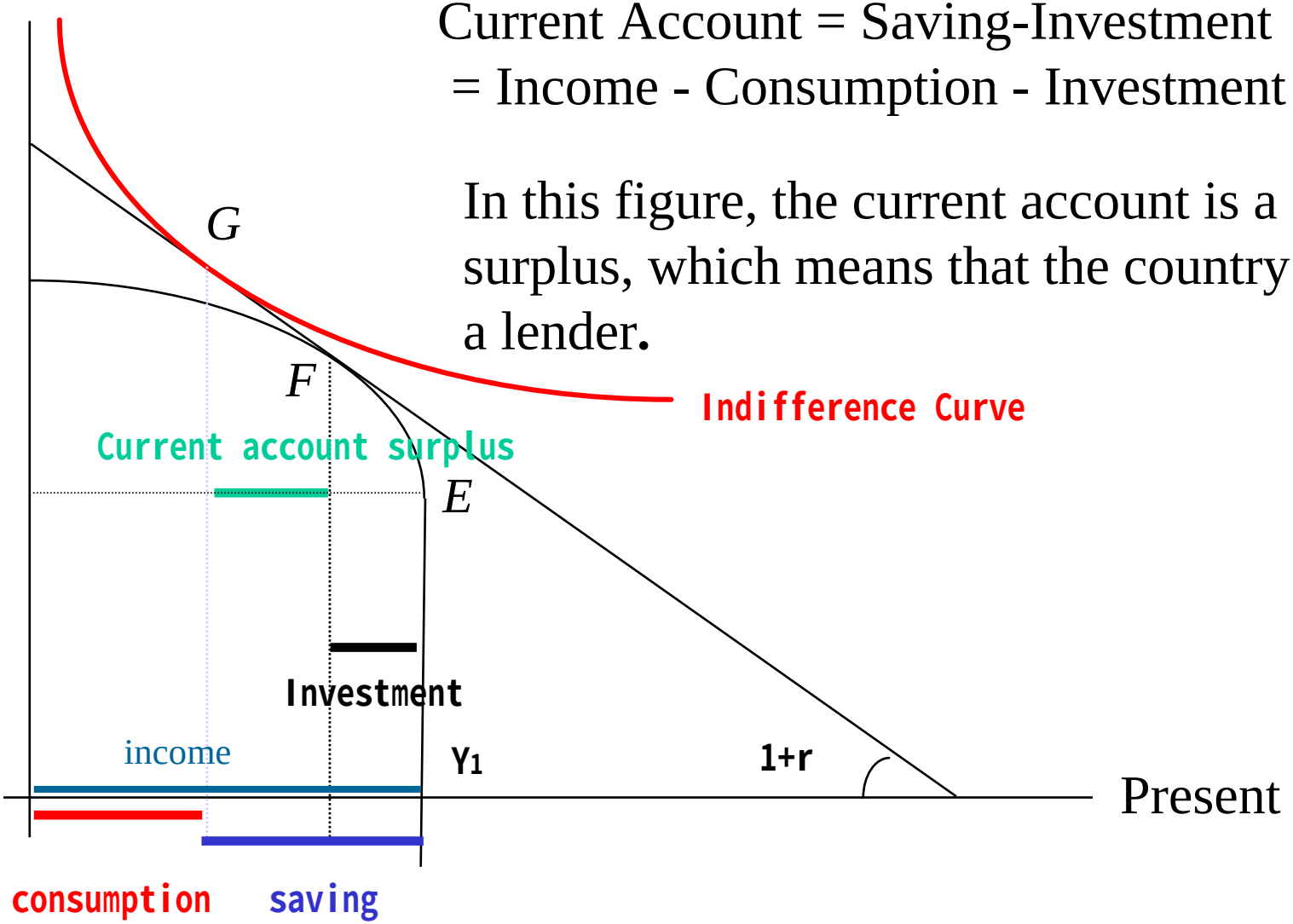
- People in a country share some preferences and technologies in common.
- We can view a country as an agent to rationally make an investment/saving decision.
- The current account of a country equals the difference between saving and investment.

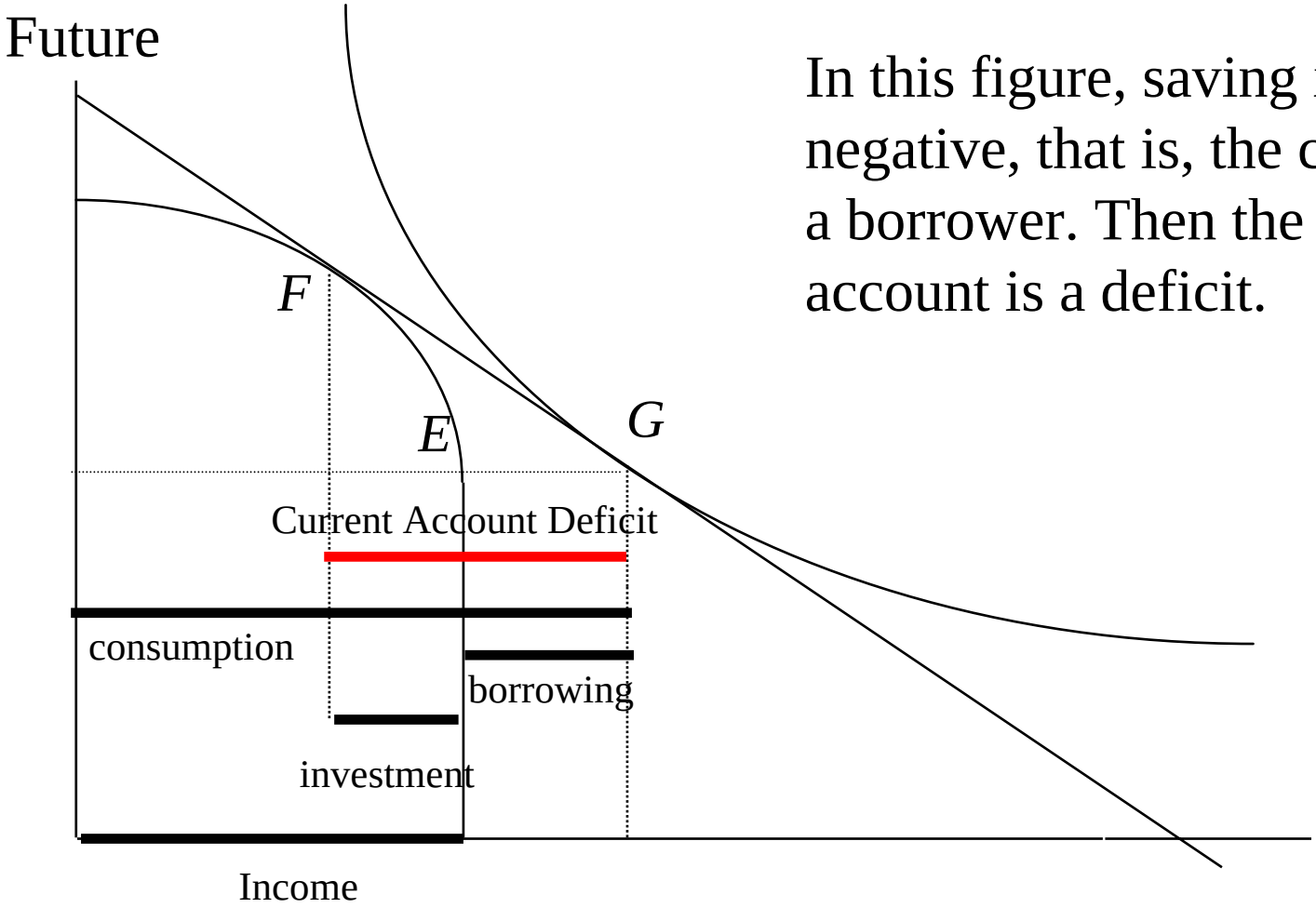
Saving = Income – Consumption

Current Account = Saving-Investment

= Income - Consumption - Investment

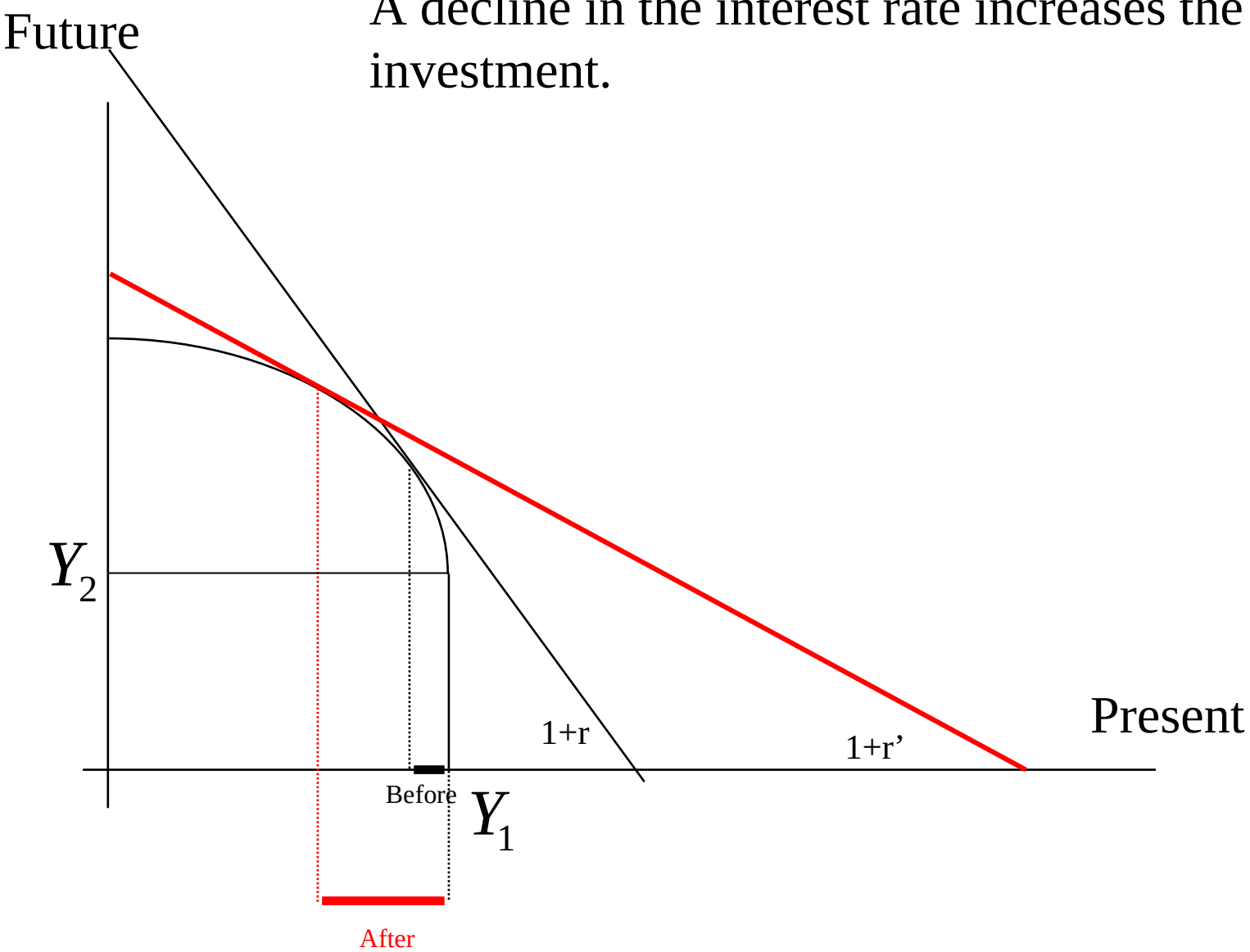
In this figure, the current account is a surplus, which means that the country is a lender.

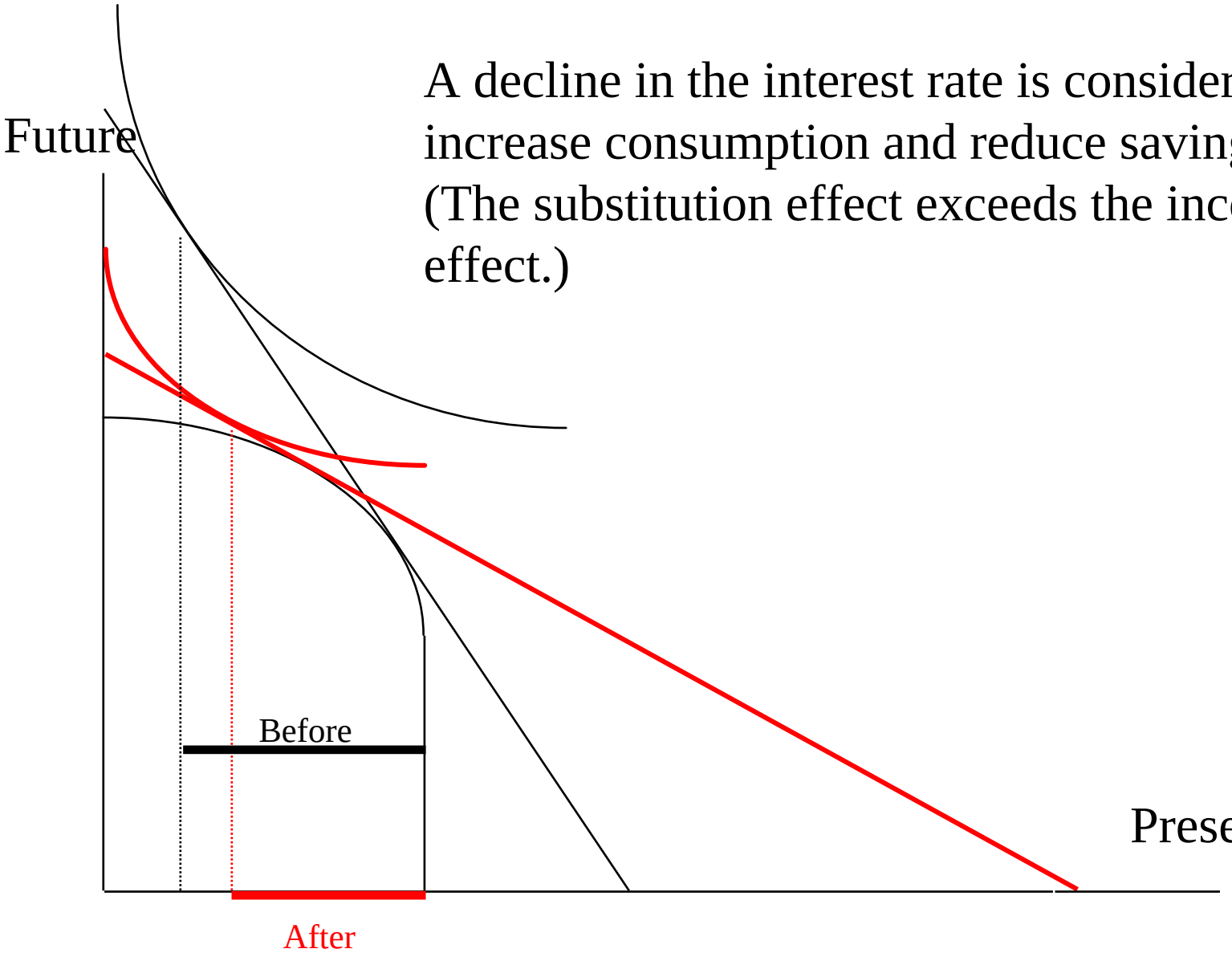




In this figure, saving is negative, that is, the country is a borrower. Then the current account is a deficit.

A decline in the interest rate increases the investment.





A decline in the interest rate is considered to increase consumption and reduce saving. (The substitution effect exceeds the income effect.)

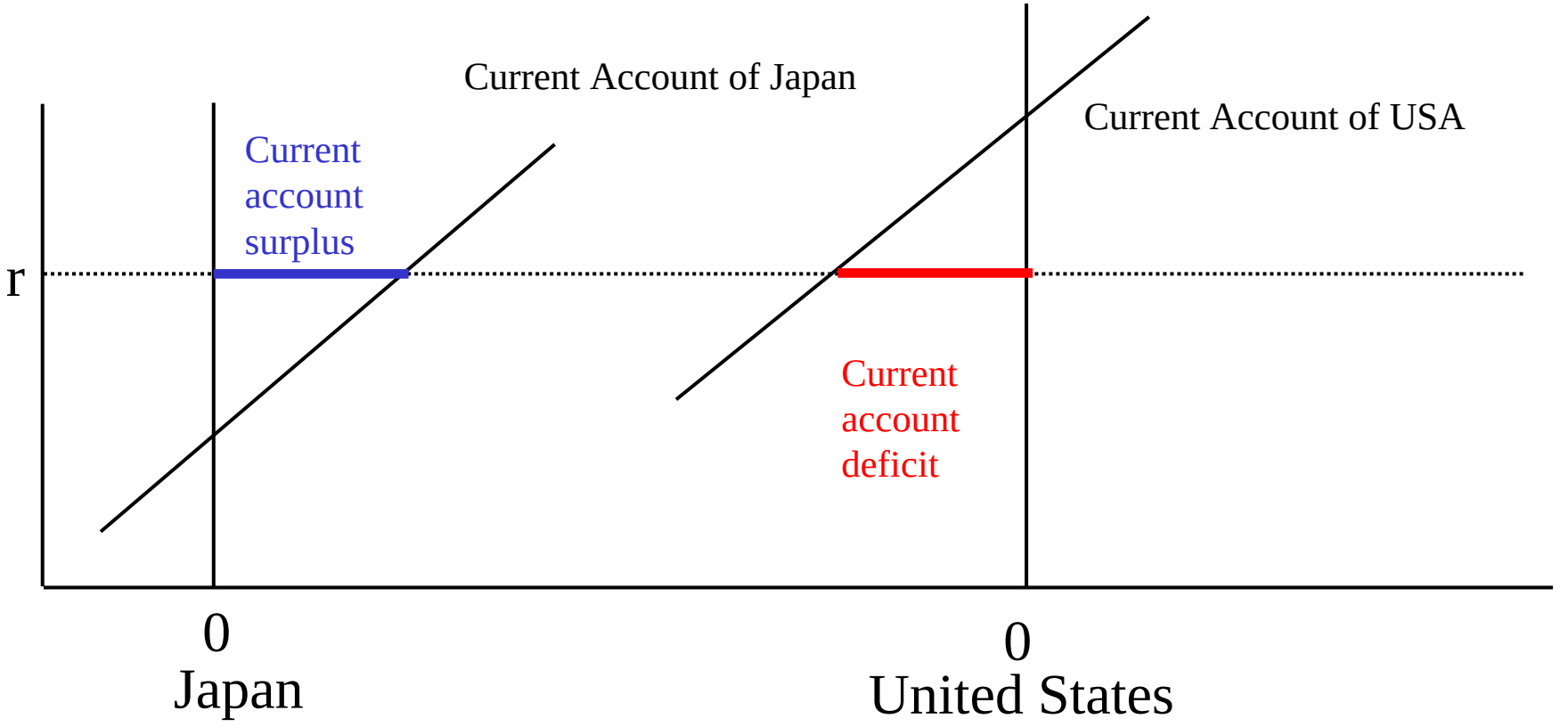
Current Account and Interest Rate

1. A decline in the interest rate increases investment and reduces saving, and then decreases the excess saving or the current account surplus (= saving – investment).
2. In other words, the current account is an increasing function of the interest rate.

Japan and the USA

1. Consider two open economies, between which capital flows with no constraints.
2. A most important example is the relation of Japan with the USA. Japan has recorded a long-run current account surplus, while the USA has recorded a long-run current account deficit.
3. A current account surplus means that the country lends to other countries, while a current account deficit means that the country borrows.
4. For Japan-US case, Japan lends and US borrows.

The Determination of the international Interest Rate (Metzler's Diagram)



The International interest rate is determined such that the excess investment of the US is just financed by the excess saving of Japan.

Questions

4. If there is a technological *progress* in US, what will happen to the world interest rate and the current accounts of Japan and US?
5. If there is a technological *slowdown* in Japan, what will happen to the world interest rate and the current accounts of Japan and US?
6. Try to explain the lost 90s of Japan in this model.

Hints: Technological progress is captured by a shift of the investment possibility frontier!