6. **The Solow Model**

Time is discrete. Assume that the representative consumer saves a constant fraction, $s \in [0,1]$, of her income. The net growth rate of variable $x$ is defined as

$$g = \frac{x_{t+1} - x_t}{x_t},$$

while its gross growth rate is given as

$$1 + g = \frac{x_{t+1}}{x_t}.$$

We refer to the net growth rate simply as the "growth rate". The transitional equation for capital is written as

$$K_{t+1} - K_t = sF(K_t, A_t L_t) - \delta K_t \tag{25}$$

where $\delta$ denotes the rate of depreciation of capital, $K$, $L$, and $A$ denote capital, labor, and the total factor productivity (technology and knowledge), respectively. In this expression, the technological progress occurs as if the labor input increases, which is termed as the **Harrod-neutral technological progress**. Let $\gamma$ denote the rate of technological progress. Then, by definition,

$$A_{t+1} = (1+\gamma)A_t$$

Therefore, the technology stock at time $t$ is a function of the technology stock at time zero such as

$$A_t = (1+\gamma)^t A_0$$

Define

$$k_t = \frac{K_t}{A_t L_t}, \qquad y_t = \frac{Y_t}{A_t L_t}, \qquad f(k_t) = F\Big(\frac{K_t}{A_t L_t}, 1\Big)$$

Dividing equation (25) by $A_t L_t$, we have the capital per *efficient* labor unit at time $t+1$ as

$$k_{t+1} = \frac{sf(k_t) + (1-\delta)k_t}{(1+\gamma)(1+n)}. \tag{26}$$

where $L_{t+1} = (1+n)L_t$. In other words, $n$ represents the growth rate of labor. The expression is equivalent to

$$k_{t+1} - k_t = \frac{sf(k_t) - (\gamma + n + \gamma n + \delta)k_t}{(1+\gamma)(1+n)} \tag{27}$$

The difference equation (27) has a unique stable steady state, $\bar{k}$, which is charactereized as

$$sf(\bar{k}) = (\gamma + n + \gamma n + \delta)\bar{k}.$$

The steady-state output per person, or the per-capita GDP, is given by

$$\frac{Y_t}{L_t} = A_t f(\bar{k}).$$

On the steady-state growth path (or **the balanced growth path**), the capital per person is constant, while the output per person grows at constant rate $\gamma$, which is the growth rate of the total factor productivity (TFP).