Modal logics of answers

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May 20, 2005

Abstract

This paper aims to apply modal logics arising from partitionistic structures to the logic of questions and answers. Taking it for granted that a partition of possible worlds serves as a representation of a context, the idea of multi-contextual situation motivates the proposal of partitionistic structures. Logical properties of answers are represented by modal operators on such structures.

1 Motivating examples

1.1 Twenty questions

Twenty questions is a two-player game to find what the answerer has in mind by posing at most twenty yes-no questions. In this example, suppose that the range of possible answers is a set of numbers from 1 to 1,000,000. Since a million is a bit less than 2^{20}, a wise questioner must be able to find the correct answer. How can she reach it?

Questioners may or may not be wise. For example, a questioner may ask "Can it fly?" to lose a precious opportunity; the question could yield no meaningful answer. Another bad way is to ask "Is the number less than 500,000?" in the first turn and get the answer yes, and then to ask "Is it more than 750,000?" Obviously, the answer is no, and he will lose a question without obtaining any information.

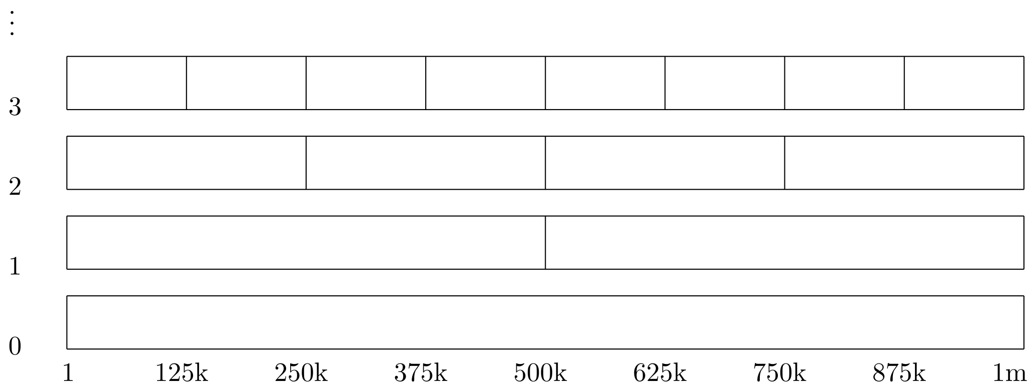
Another poor way is to repeat a question. Suppose the question "Is the number less than 500,000?" in the first turn gets the answer yes. Repeating the same question "Is the number less than 500,000?" would obtain the answer yes, but it does not help the questioner to squeeze the range of possibilities any more.

Now, it is natural for logicians to ask the following questions. What logical properties do such poor ways of asking questions bear? Can they be characterized?

Let us return to focus on a wise questioner. A systematic strategy is to cut the range in the half in each round. For example, asking "Is the number less than 500,001?" in the first turn submits a binary partition of the original range:

from 1 to 500,000 for the one hand and from 500,001 to 1,000,000 for the other hand. The question will get an answer "yes", if the correct answer is 370,247. Then, in the second round, she could ask, "Is the number less than 250,001?" and will get an answer "no". Then, she could continue, "Is the number between 250,001 and 375,000?" to obtain the answer "no," and so on.

In such a strategy, each yes-no question in the sequence poses a binary partition of the range of possible answers. Since the range is cut in a half in each round, the question at the next stage will cut the remaining in the half again. Thus, a series of questions following the systematic strategy can be represented as nesting partitions.

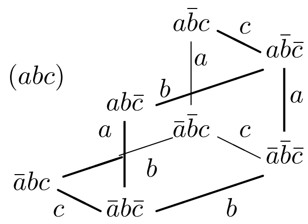


We can ask ourselves again: is there any characterization of systematic strategies for this game? In particular, can any modality capture their logical properties?

1.2 Muddy children

The muddy children puzzle is another game where questions and answers play the central role. Let us consider the case of three players, a, b, and c. Every player has a playing card and can see the card of all the players but herself. An observer comes and says, "at least one of you has a red card," and then asks: "do you have a red card?" Suppose that everybody has a red card. Each player sees the others', and says "I don't know" in the first round. The same happens in the next round. Suddenly in the third round, everybody says, "Yes, I have a red card." What information is obtained by the iterated utterances of "I don't know"?

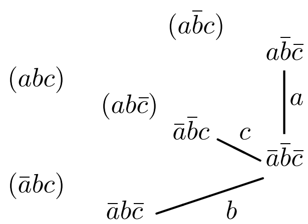
The standard analysis of the muddy children puzzle with possible worlds also casts a light on the use of possibilities in information. The situation with the announcement "at least one of you has a red card" is described using possible worlds as follows:



In the diagram, each node represents a possible world, and each line with a label a, b, or c represents an indiscernibility relation. a cannot distinguish $ab\bar{c}$ and $\bar{a}b\bar{c}$, for example, because a cannot see her own card.

The possibility that all players have a non-red card (represented by abc in the diagram) is eliminated by the initial announcement. Let nodes in parentheses represent eliminated possibilities. The indiscernibility relations connecting an eliminated possibility to others also disappear. With the possibility which is incompatible with the announcement being eliminated, any player should reason that she herself must have a red card, if she saw there is no other player who has a red card.

The answers "I don't know" in the first round eliminate the possibilities that each player sees that no other players have a red card. The reason is that, if any player saw that there is no other player who has a red card, she should have already know that she herself must have a red card at the moment of the initial announcement. Therefore, the answers "I don't know" imply that every player sees at least one other player who has a red card.



The apparently same answers "I don't know" in the second round eliminate the possibilities in which two players have a red card. The resulting diagram has the only surviving possibility $\bar{a}\bar{b}\bar{c}$, i.e., the possibility that every player has a red card.

$(a\bar{b}c)$

$(a\bar{b}\bar{c})$

(abc)

$(ab\bar{c})$

$(\bar{a}\bar{b}c)$

$\bar{a}\bar{b}\bar{c}$

$(\bar{a}bc)$

$(\bar{a}b\bar{c})$

The point of the puzzle is that the apparently-same answer "I don't know" eliminates different possibilities at each round so that only one possibility survives after the final round. Are there any differences between logical properties of the answers in the final round and those beforehand? Can any modalities represent the differences?

2 The logic of questions and answers: a previous treatment

Logical investigations of questions and answers are not new. The area of logic of questions and answers has developed since the 1950s. Informal analyses in the research in the 1950s aim:

1. To clarify logical relations between a question and its answers;
2. To formulate logical relations (equivalence and implication, for example) among questions in a context; and
3. To classify answers to a question into subcategories according to their logical properties in a context, and characterize those subcategories.

In particular, Belnap-Steel [3] develop a logical theory of questions and answers. Their central task is twofold: to construct a formal language to describe the roles of questions and answers in linguistic communication on the one hand, and on the other hand to conceptually analyze erotetic notions. They first distinguish interrogatives as sentences in an everyday language from questions as abstract objects[1].

In their analysis, a question is represented as a combination of a presupposition and a set of mutually incompatible and exhaustive answers. Each of the mutually incompatible and exhaustive answers is called a *direct answer*. A direct answer to a question is a statement to answer the question completely and just completely, i.e., gives information just enough to meet the questioner's expectation. A direct answer leaves no further inquiry for the questioner.

It is not assumed that every question has at least one true answer, although it might be expected to hold. Instead, a question is called *foolish* when it has

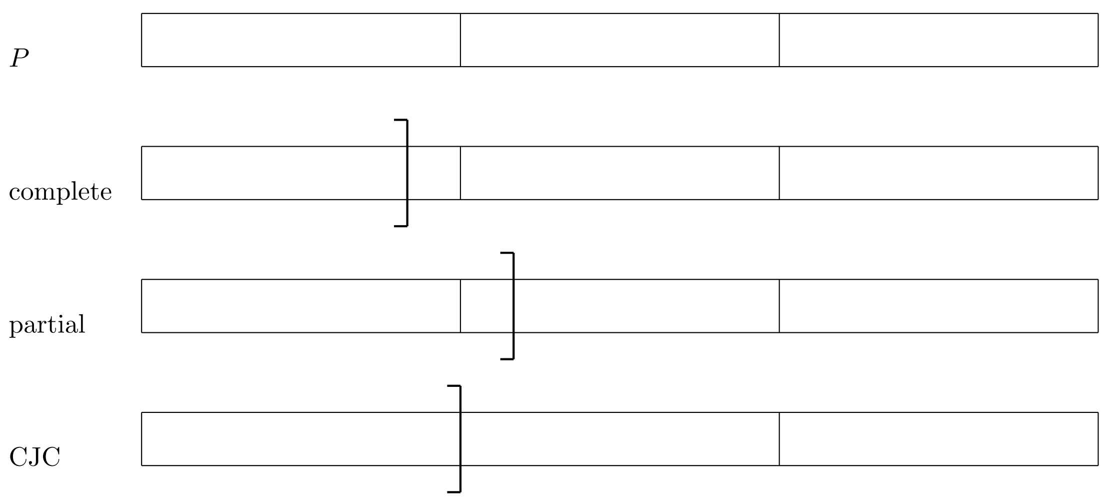
[1]The distinction is analogous to declaratives and propositions, multi-multi correspondences exist between them.

no possible direct answer. A question is *meaningful* when it is not foolish. A question is called *trivial* when it has only one possible direct answer, or there is no alternative answer. Answers to a question are sorted out with respect to its direct answers.

- An answer is *complete* if its truth set is non-empty and it is a subset of a direct answer.
- An answer is *partial* if its truth set is a superset of a direct answer.
- An answer is *foolish* if its truth set is disjoint from the presupposition set of the question[2].
- An answer is *complete and just complete* if its truth set is a direct answer.
- An answer is *informative* if its truth set is a proper subset of the presupposition set.

A similar set of notions can be introduced to speech acts of a wider range by taking a request as a set of direct responses.

The relationship among the notions of complete, partial, and complete and just complete answers can be illustrated in the following picture. Those notions are dependent on the receiver's expectation, which is represented by a partition P. A partial answer is a superset of a member of P, and a complete answer is a subset of a member of P. A complete and just complete answer coincides with a member of P.



[2]Thus, a foolish question must have a foolish answer.

2.1 Twenty questions with logic of questions and answers

Now we have an analyzing device in hand.

2.1.1 A foolish question

The question “Can it fly?” in the initial example is foolish. It could yield no meaningful answer.

Another foolish way is to ask “Is the number less than 500,000?” in the first turn and get the answer yes, and then to ask “Is it more than 750,000?” The second question is foolish, as the range of possible answers does not contain the correct answer at all, i.e., all possible answers to the question are foolish.

2.1.2 A trivial question

Repeating a question is another poor way, because repeated questions turn out to be merely trivial. Their true answer does not eliminate any possibilities, i.e., it is not an informative answer.

2.1.3 Complete and partial answers

An ideal strategy of the questioner is to systematically narrow down the range of possibilities. On the other hand, the answerer may keep the range of possibilities as large as allowed by the rules of the game. Thus, the answerer is considered to be poor if he gives the following sorts of answers to the question “Is the number less than 500,000?”

- “The number is between 1 and 700,000” or “The number is between 300,000 and 600,000.” Both give neither “yes” nor “no” to the given question, and he violates the rules of the game. Both answers are informative. The former is a partial answer, while the latter is not, to the given question.
- “The number is between 300,000 and 400,000” is a complete answer to the given question. The answer is indeed true and informative, if the correct answer is 370,247. It spoils the game, however.

2.2 Muddy children with logic of questions and answers

The same conceptual device can describe the difference between the answers in the final round and the other answers beforehand. Only the former is complete and just complete to obtain the intended answer “Yes, I have a red card.” The answers in the latter case are merely partial answers. They are not sufficient to specify either of the intended answers, “yes” or “no”.

3 Multiple contexts with questions

Questions often can be answered in various ways. A single interrogative sentence may pose more than one question at the same time.

Example 1 *Twenty questions. The answerer can give a true and complete answer if the questioner gives a meaningful question. It is the best legitimate way for him to give just "yes" or "no", either of which is a complete and just complete answer.*

Example 2 *Muddy children. The interrogative "Do you have a red card?" can be answered at least by the two different sets of legitimate answers. The first set consists of the answers "Yes, I have a red card" and "No, I do not have a red card." The second set consists of the answers "I know that I have a red card," "I know that I do not have a red card," and "I do not know whether I have a red card."*

The setting of the muddy children puzzle entertains the ambiguity in addition to veridicality of epistemic modalities. It does not count the first set as a legitimate set of answers in the first several rounds.

Obviously, asking a question for information is not straightforward. It depends on contexts, including surrounding situations, the previous knowledge, and the answerer's knowledge of the questioner's knowledge. Compare the following two examples for contrast.

Example 3 *Muddy children with background knowledge on the colors of the suits. Suppose the players have background knowledge about standard sets of playing cards. If somebody sneaks in the room and declares "all of you have a diamond," every player can answer "yes" to "Do you have a red card?"*

Example 4 *Muddy children without background knowledge on the colors of the suits. Suppose there is a player who does not know that diamond is a red suit. He cannot conclude that he has a red card even with the announcement "all of you have a diamond."*

Economy of communication matters, too. In our everyday life, it is considered to be desirable to answer to a given question according to the expectation and background knowledge of the questioner.

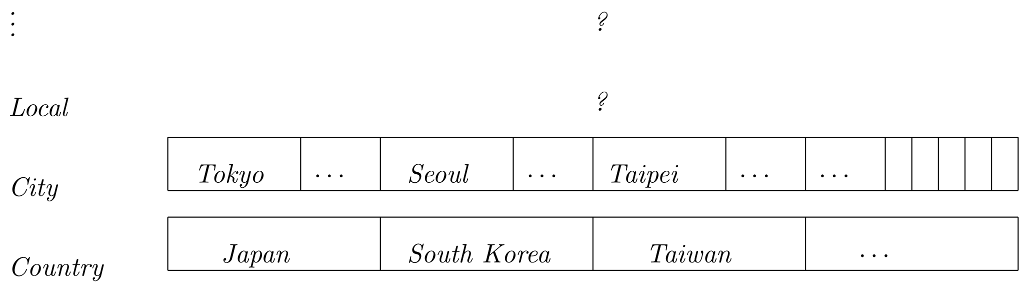
Example 5 *Asking where the Sample Gates are in Bloomington in August bears the information that the questioner is new on Indiana University campus as well as that she needs to know where the gates are. Utilizing the information, a wise answerer tries to answer the question mentioning nothing that the only old residents should know. An answer "The Sample Gates are at Kirkwood and Indiana" would bring little information for those who have no idea where Kirkwood is.*

Nevertheless, the same answer would be trivial for old residents, too, if they pose the same question. They should know it already. What they might expect is, for example, the exact street address of the gates.

Example 6 *"What treatment is needed for a patient of typhoid?" Suffering patients would be satisfied with answers such as "Antibiotic shots will work in addition to hospitalization for fluid therapy," and would not care much about more information such as the kind of antibiotics and the recommended amount of the shot. Rather than listening to a long answer, they want to go to a hospital as soon as possible. Even a simpler answer "hospitalization" suffices.*

Medical providers, however, would need more detailed information to save the life.

Example 7 *There are several ways to answer, when I am asked "Where are you from?" To those who I meet in Bloomington for the first time, I would say "From Japan." Usually they continue to ask "Which city?" I add "From Tokyo."*



To my new friends in Tokyo, however, I would answer them with local information. I may mention railroad stations near my home. If I answered "From Japan" to them, they would be perplexed. They already know that I am from Japan, and see no point in my saying "From Japan." The answer is trivial, and they will suspect that there might be hidden assumptions or an unexpected intention.

Such local information will not work on Hoosiers. They cannot get any idea where I am actually from. When the receiver is not able to figure out in which country the station or the landmark I give is, the set of possible worlds in his view should be classified so that, for each country, there are possibilities that the station exists there.

⋮				
Station	*Japan*	*South Korea*	*Taiwan*	⋯

Thus, it seems natural to assume that there are many layers of legitimate answers to a single question, in general. Some layers work, and the others do not. It depends on contexts.

The presentation of the logic of questions and answers in the last section focuses on only a single layer of answers. Belnap and Steel take such multiple layers of questions into consideration. They conceptually separate interrogative sentences and questions so that a single interrogative in an everyday language can bear multiple questions. To model situations involving questions and answers in an everyday language by possible worlds, it seems better to prepare with multiple layers of possible answers.

4 Proposal

Can the modal approach do anything in logic of questions and answers? Yes.

4.1 Proposal

My proposal for the logic of questions and answers is this. The main idea of this thesis, partitionistic elimination, is that complete and just complete answers should be taken as the only legitimate answers in each occasion. Focusing on complete answers leads to variants of logical omniscience; to avoid it, we need specification of a legitimate set of complete and just complete answers. It can be semantically formalized with multiple partitions, and the resulting modal logics are non-monotonic so that they avoid logical omniscience.

4.2 Possible world formalization

First, possible world semantics can be used to formulate the concepts informally described in the last section. A partition corresponds to a set of direct answers, or a question. A set of partitions describes the set of acceptable ways of answers.

Second, each notion of the logic of questions and answers has a corresponding modal operator as its formal counterpart.

4.2.1 Complete and just complete answers

In applications to the logic of questions and answers, the unary modality ∇, argued in Murakami [7], is interpreted as the indicator of complete and just complete answers[3]. Its intuitive reading is this: when ∇A holds at world w in model M, a message A serves as a complete and just complete answer according to the receiver's expectations. The logic of ∇ calculates logical relationships among sentences which meet the receiver's expectations in the situation of such a message transmission.

Example 8 *Let us return to the example "Where are you from?" when the hearer is a person who meets me for the first time in Indiana. The formal language for the situation has atomic propositions* `Japan`*,* `Taiwan`*,* `SouthKorea`*,* `Tokyo`*,* `Seoul`*,* `Taipei` *among other propositions involving countries and cities she knows[4]:* `Japan` *stands for "I am from Japan", for example. Each possible world represents a combination of a country and a city. Since the language here represents the situation that she knows the relationship of each city and each country, there are only combinations reflecting the real situation; i.e.,* $\{\texttt{Seoul}, \texttt{Korea}\}$*,* $\{\texttt{Taipei}, \texttt{Taiwan}\}$*, etc. One of which is the actual world that represents where I am from. Assuming that one's home country uniquely exists, we can pick up a partition of the set of possible worlds. She is furnished with two levels of answers, the country level and the city level. The city level is a subpartition of the country level. In such a situation, she will get complete and just complete information from my answer "From Tokyo".*

Nevertheless, she would be perplexed by my answer "Ichigaya" if she does not know where it is. Adding `Ichigaya` *to the language splits possible worlds: then in the new model, the set of epistemic/ doxastic possible worlds includes those each of which is represented by a set of propositions,* $\{\texttt{Ichigaya}, \texttt{Tokyo}, \texttt{Japan}\}$*,* $\{\texttt{Ichigaya}, \texttt{Seoul}, \texttt{Korea}\}$*,* $\{\texttt{Ichigaya}, \texttt{Taipei}, \texttt{Taiwan}\}$*, for example. The latter two do not reflect the real world, as Ichigaya is in fact the name of a station in Tokyo. Unless she is well-prepared with the level of answers, she would not get complete and just complete information from such a message. Moreover, even if she guesses that Ichigaya is some area in a city, she will not be able to infer that I am from Tokyo, unless she does know in which city it is, This situation is represented by partitions which are not nested in each other.*

Example 9 *Negative question. Some linguists oppose the partitionistic approach by pointing out that those two interrogative sentences are taken as a same question in every context in the partitionistic approach.*

- *Is a whale mammal?*
- *Isn't a whale mammal?*

[3] ∇ allows several readings, such as an imperative reading.

[4] She may not know some cities in those countries. Adding their names would yields the same situation as one with additional local information with no connection to other levels.

It is not a problem. We should consider that multi-multi relation between interrogatives and questions as well as declaratives and propositions. Then, we can say that those interrogative sentences represent the same question.

A possible objection is that, if equivalent, differences of usage between two interrogatives should be reflected as failure of logical equivalence between them. The answer to such a question is, however, that it is not necessary. Logical equivalence among questions should be not supposed to reflect pragmatic aspects of our usage of questions. Even for declaratives, the notion of logical equivalence is much coarser than the identity of meaning. Consider mathematical statements. The sentence that 1+1=2 does not mean the same as the sentence that Fermat's theorem holds.

Example 10 *Negative answer.*

- *Who passed the exam?*
  - *John.*
  - *Not John.*

The second answer may or may not make sense. Multi-partition settings can specify cases in which the receiver expects it. Consider the scenario: Mary just transferred to a new school. She became acquainted with John, but she has not talked with the other classmates yet.

In fact, negative answers are usually partial. If Mary could identify all her classmates, the answer "Not John" would be equivalent to the disjunction of all the classmates except John. The problem is that the question does not exactly specify the range of possible answers.

5 Modal logic on multi-partition

The possible world formulation easily gives truth conditions for information-related modalities: For example, where $\star A$ is read "A is informative" (in the eliminative view), the truth condition will be $M, w \models \star A$ iff there is w' such that $M, w' \not\models A$. Another modal sentence $\star A$ whose intended reading is that A is a complete and just complete message can be interpreted as: $M, w \models \star A$ iff for any w', Rww' if and only if $M, w' \models A$.

The notion of informativeness in the logic of questions and answers is also implemented in terms of possible worlds: A message is informative with respect to a context if all possible worlds in an element of the partition are compatible with it.

The notion of complete and partial are restated as:

- A message is complete with respect to a context if only possible worlds in an element of the partition are compatible with it
- A message is partial with respect to a context if all possible worlds in an element of the partition are compatible with it. (So the modality '$\cdots$ bears partial information' should behave as S5).

Also the notion of complete and just complete information: A message is just informative with respect to a context if all and only possible worlds in an element of the partition are compatible with it.

5.1 Semantics and formal results

See Murakami [7] for details of formal definitions and results, including a completeness proof.

A p-frame is $\langle W, \mathtt{P}\rangle$, where W is an non-empty set, and $\mathtt{P}$ is a set of partitions on W, i.e., each $P \in \mathtt{P}$ is a partition of W. A valuation v on p-frame $F = \langle W, \mathtt{P}\rangle$ is a function which assigns a subset of W to a propositional letter. A p-model on a p-frame $F = \langle W, \mathtt{P}\rangle$ is $\langle W, \mathtt{P}, v\rangle$, where v is a valuation on F.

Let $[x]_P$ stand for the member $U \in P \in \mathtt{P}$ such that $x \in U$.

The truth condition for ∇ is: $M, x \models \nabla A$ iff there is $P \in \mathtt{P}$ such that $[x]_P$ is the truth set of A in M; and the truth condition for $[\forall]$ is: $M, x \models [\forall]A$ iff $M, w \models A$ for every world $w \in W$. When ∇A holds at x in M, a *witness* for ∇A at x in M is $P \in \mathtt{P}$ such that $[x]_P \in P$. It may not be unique.

The logic of the class of all p-frames is axiomatizable and decidable. In fact, an axiomatic system $L([\forall], \nabla)$ is sound and complete with respect to the logic.

1. PC
2. S5 for $[\forall]$
3. $[\forall](p \to \nabla p) \vee [\forall](p \to \neg\nabla p)$
   (Partition axiom)
4. $\nabla p \to p$ (Reflexivity axiom)

Logics of some other class of p-frames are also axiomatizable. For example, the logic of the class of all p-frames with its partition set being a singleton is axiomatizable with an additional characterization axiom: $(\nabla p \wedge \nabla q) \to [\forall](p \to q)$.

Nevertheless, not all logics on partitions are axiomatizable. Consider a subpartition relation on a set of partitions; i.e., a partition P is a subpartition of P' iff for any $s \in P$ there is $s' \in P'$ such that $s \subseteq s'$. A p-frame is *linear* iff its partition set is linearly ordered with the subpartition relation. It is shown then that there is no axiom characterizing the class of all linear p-frames.

6 Complete answers versus complete and just complete answers

Philosophers repeat that human beings' ability to reason is incomplete. An ordinal person would not see what happens even if there does exist total evidence indicating a symptom for those who have been trained. Nevertheless, it is not just a matter of reasoning ability. We need to be trained not just on how to reason from a given data but also how to *obtain data* and how to relate with other information. This is why I advocate to use partitionistic elimination.

6.1 ∇: some formal properties

See Murakami [7] for details.

- Non-monotonic: $A \to B$ does not infer $\nabla A \to \nabla B$.
- Non-antitonic: $A \to B$ does not infer $\nabla B \to \nabla A$
- $\nabla(A \vee B)$ is implied by $(\nabla A \wedge \nabla B)$, but not vice versa.
- $\nabla(A \to B)$ implies neither $\nabla\neg A$ nor ∇B.

In the class of frames with the singleton partition set, $\nabla(A \to B) \wedge \neg[\forall]A \to \neg\nabla A$ is valid.

6.2 Adequacy with respect to intended applications

Let us examine some utterances to see if the notion of complete information can be applied to these cases.

Example 11 *Implicative claims illustrate the case of giving complete and just complete information:*

- *If Alice went to the party, so did Bill.*

Provided that he is sincere to give all the information he has, the speaker does want to commit himself to claim neither of the following:

- *Alice did not go to the party.*
- *Bill went to the party.*

Example 12 *Another example of complete and just complete message is a promise:*

- *If it is not raining, we will go to the zoo.*

Notice that $\nabla(A \to B)$ implies neither ∇A nor ∇B, and $\nabla(A \to B)$ implies $\neg\nabla A$. If an utterance of the promise bears a complete and just complete information, the utterer does not want to claim that it is raining, nor he claims that we are going to the zoo at the moment of the utterance[5].

Observations from the examples seem to lead to *maximal informativeness conjecture*:

> Utterances to conduct a speech act bear just enough information. Merely complete information is not adequate.

[5] A similar phenomena is observed on imperatives.

6.3 Previous usage of complete messages

While I would propose to take complete and just complete answers as only sufficient answers in each occasion, many authors focus on complete answers as paradigm of informative answers. Developing the theory of information content, for example, Dretske and Barwise exclude the problem from logic: incomplete reasoning ability. Dretske [4, p.14] analyzes:

> Once something happens, we can take what did happen as a reduction of what *could* have happened to what *did* happen and obtain an appropriate measure or the amount of information associated with the result.

Furthermore, Dretske characterizes his notion of *information content* by non-triviality, a special case of counterfactuality in a non-temporal context:

> A signal r carries the information that s is F iff the conditional probability of s's being F, given r (and k), is 1 (but, given k alone, less than 1) [4, p. 65]

Notice that the message r can be taken as a complete answer to a question which expects s's being F. The message "John and Mary will come to the party this evening" will bring the information that Mary will come to the party. It is no problem with such a simple example. Nevertheless, the message "Sample gates are at Kirkwood and Indiana" brings the information where it is no matter what background knowledge its receiver has.

Imperfect reasoning ability can play the role of excuse with while we want make the formulation much closer to everyday phenomena[6].

The idea of partitionism can be applied to an analysis of the possible world characterization of information content (Barwise [1]):

> To a person with prior knowledge k, r being F carries the information that s is G if in all the possible worlds compatible with k and in which r is F, s is G (and there is at least one possible world compatible with k in which s is not G).

The eliminative view, extracted in the inverse information principle, focuses on the note inside of the parentheses, which stipulates informativeness.

Moreover, the characterization of information content can be again restated in terms of partitionism, if the following assumptions are accepted:

1. The body of prior knowledge k agrees with the presuppositions of the context $C(k)$.

2. What carries information is a message. The message, whose content is that r is F, decreases the presupposition set by eliminating possibilities incompatible with it. Let $t(F)$ be the subset of $C(k)$ where 'r is F' holds.

[6] Barwise's idea of constraints has the same point. Comparison and correspondence between constraints and multiple partitions will be a good project beyond this thesis.

3. The information that s is G in the definition of information content represents a piece of a focus in the partitionist characterization. The condition in the parentheses stipulates that the focus partition is not trivial. Let $t(G)$ be the piece of the focus partition, or the proper subset of $C(k)$ where 's is G' holds.

Therefore, their approach to information contents takes complete messages as paradigmatic cases. It assumes that the receiver of the message can infer any consequences in the context. In general as seen in Example on page 8, however, getting local information about Ichigaya station is not enough to conclude that I am from Tokyo.

Such problematic situations require an amendment. There are at least two possible ways: either to control the set of possibilities or to manage the receiver's inference ability.

The former is seen in Example 8 on page 10. In such an approach, epistemic possibilities may or may not reflect the actual world. These epistemic possibilities may not be simply taken as representations of the actual world, in particular, when changes of the world are described by any sort of possible worlds. The question how to associate those two sorts of possible worlds will require further investigation beyond the thesis.

An alternative following the latter amendment is the following partitionistic characterization of information content:

> In the context $C(k)$ represented by k, the message r being F carries the information s being G is asked, if the focus partition is not trivial, and if $t(F) \subseteq t(G)$

In light of the notions of logic of questions and answers, it becomes clear that the definition characterizes the notion of a complete message, not a message with just enough information. It fails to exclude messages with too much information. No problem, as far as the interest is in how information flows. It is problematic only if our interest extends to the treatment of information excess.

6.4 Only: some formal properties

In fact, the logical property of complete information is reflected by the "only" operator. Gargov et al. [5] argue the logic of [ext] and [only], whose truth conditions are:

$$\begin{array}{lll} M, w \models [\text{ext}]A & \text{iff} & \text{for every } w' \text{ whish is not } Rww',\ M, w' \models A \\ M, w \models [\text{only}]A & \text{iff} & \text{for every } w', \text{ if } M, w' \models A \text{ then } Rww' \end{array}$$ [7].

[only] has the following properties.

- Antitonic:

$$\frac{A \to B}{[\text{only}]B \to [\text{only}]A} \tag{1}$$

[7] It is also suggested in van Benthem [8]. He proposes to read $[\text{ext}]A$ as "$\neg A$ is strongly permitted" from the intuition (von Wright [9]) that strong permission of A or B should be equivalent to the conjunction of strong permission of A and of strong permission of B.

- [only]$(A \vee B)$ is equivalent to the conjunction ([only]$A \wedge$ [only]B) [De Morgan property]

Thus, [only]$(A \rightarrow B)$ is equivalent to [only]$\neg A \wedge$ [only]B.

Nevertheless, as [only]$(A \rightarrow B)$ is equivalent to [only]$\neg A \wedge$ [only]B, if [only] is used to represent insistence, upon the initial insistence he should insist both. Thus [only] should not be used for this representation purpose.

On the other hand, if ∇ represents a complete and just complete message, the initial claim entails only that he does not give complete and just complete information that Alice went to the party. This result is better than the [only] case.

7 Classification of partial answers

There are three classes of partial answers. First, a loose reading of the definition of a partial answer allows such a situation that an answer is partial but not informative. The second class consists of answers which are partial and informative: a partial message in the first sense is such a message that is compatible in all the possible worlds in a direct answer or a piece of the partition, and no specification about the other possible worlds outside of the piece.

Third, as we use often in everyday situations, some partial answers are indeed disjunctions of complete and just complete answers. The category of partial messages is more specific. A partial message in the third sense is such a message that is a disjunction of direct answers, or compatible with all the possible worlds in some pieces of the partition, and incompatible with all the possible worlds in the remaining pieces.

Of course, partial messages perplex us. It does not seem to bring adequate information anyway, but which category do we tend to take to be natural? Consider this example: the question "which do you like, a soup, a salad, or an appetizer?" sets a context which can be represented by a partition with three pieces labeled with SOUP, SALAD, and APPETIZER. Non-complete answers can be:

- Soup or salad.
- Soup or caesar salad.
- Misoshiru or caesar salad.

It can be the case that the first falls in the third category, the second falls in the second, and the third does not fall in either.

Formal counterparts illustrate logical differences among the three notions. The first class, a standard S5 operator serves as a formal counterpart of logical properties of answers which are partial but may or may not be informative. A deliberative *stit* operator[8] [dstit] exactly corresponds to the second class of

[8] Belnap et al. [2] and Horty and Belnap [6] give conceptual analysis. An axiomatic system and its decidability are argued in Xu [11] and Xu [10].

partial and informative answers[9] The third class of unions of complete and just complete answers is also logically characterized by a new modal operator [pure]. Every axiom of the (single-agent) [dstit] system is provable in the system of [pure]. On the other hand, the counterpart of the purity axiom in [dstit] language is not valid in [dstit] frames. Thus, the [dstit] logic is a sublogic of the [pure] logic. See Murakami [7] for details.

Comparing their logical properties, the first and second subnotions, which rather seems to fit our intuition more than the third, still excludes the third category. Adequacy of the third sort of answers depends on whether such information excess can be accepted; if the questioner can serve misoshiru and caesar salad, such an answer will cause no problem, but it specifies the answerer's preference more than expected in the way of questioning.

8 Summary

The idea of partitionistic elimination is applied to the logic of questions and answers. The proposal is to construct possible world semantics of operators complete, partial, and complete and just complete messages. The basic logic of those modal operators are axiomatizable. There are three subnotions of partial answers whose logical behavior differs.

Complete and just complete messages should be a paradigm of messages in efficient information inquiry. A paradigmatic use of merely complete messages leads to the logical omniscience problem. Behind the idea of complete message, there is an implicit assumption of the receiver's ability of reasoning. To avoid these sort of paradoxes of epistemic logic, it is necessary to explicitly describe acceptable messages instead of the idealistic assumption. The proposed method can model the receiver's reasoning more accurately. A set of partitions of a possible world set will serve as a representation, and we can calculate relationships among sentences with respect to the receiver's expectations and background knowledge.

Technical aspects open for further investigations. An intended interpretation of a linearly ordered set of partitions is a situation with multiple contexts where a message bears information in various levels of details, as seen in everyday life where ambiguous messages often appear. Impossibility of characterization of the class of frames a linearly ordered partition set, while being formally interesting, looks despairing for the application.

It seems necessary to extend either semantics or language, or even both, to characterize such a notion. Nevertheless, it is still open whether a similar class of frames with a linearly ordered partition set is characterized in a system of any language.

[9] See Murakami [7].

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